Differential Configurational Entropy in Nonlinear Self-Similar Optical Rogue Wave

Pooja, Sukhdeep Kaur and Medha Devgan

Department of Physics, Guru Nanak Dev University, Amritsar, Punjab, India.

E-mail:poojaphy19@gmail.com

(Received: July 25, 2025, Revised: Aug 16, 2025, Accepted: Aug 26, 2025, Published: Sep 30, 2025)

Abstract: Information theory is fundamental to understanding modern communication and computation, with applications spanning chemistry, biology, high-energy physics, computer science, and condensed matter physics. Differential Configuration Entropy (DCE), a classical form of information theory, quantifies spatial complexity by measuring bounded functions logarithmically in Fourier space. It calculates the precise information required to define a function's spatial shape relative to a set of parameters. We calculate the DCE for rogue waves in a tapered graded-index optical waveguide, modeled by an inhomogeneous nonlinear Schrödinger equation. The results show a specific rogue wave width at which the DCE is minimized, indicating the point of maximum information compression in the Fourier modes that describe the wave's spatial structure. A lower DCE signifies a more concentrated wave solution and greater accuracy in predicting its localization. This minimum DCE corresponds to the optimal wave width where the information about its spatial shape is most compressed into its momentum modes.

Keywords: Differential Configurational Entropy, Optical Rogue Waves, Tapered Graded-Index Waveguide, Nonlinear Schrödinger Equation, Spatial Complexity.

I Introduction

A wave with ubiquitous nature which appearers from nowhere and leaves no trace, also known as freak waves, monster waves, killer waves are the rogue waves which forms in the ocean [1,2]. The wave posses high amplitude pulses, even higher than average wave crest [1,2]. Now days the study of these waves in other physical systems such as nonlinear optics, Bose-Einstein condensates (BEC), superfluids [3], superfluids [4,5] and capillary waves is also being pursued [5,6]. In recent years, rogue wave in optical fibers is of great interest to Physicist. The first optical rogue wave was observed by Solli *et. al.*in supercontinum generation in fibers and in the context of optical turbulance [7].

Mathematically, these waves are well described by rational solutions of nonlinear Schrödinger equation (NLSE), which are localized in both space and time. NLSE being an integrable system, a hierarchy of its higher order rogue wave solutions can be obtained by using Darboux transformation [7]. After significant amount of theoretical and numerical studies, a thrust in the direction of their controllable experimental observation is catching pace. First ever work in this direction was done by Solli *et. al.*. They showed the existence of rogue waves in nonlinear fiber optics, and the concept of optical rogue wave was introduced as a counterpart of the oceanic rogue waves. These waves were found to be generated infrequently from initial smooth pulse, resulting from the power transfer seeded by small perturbation. The experimental ability to dilate the temporal duration with group velocity dispersion, played a significant role in their observation. Rogue wave dynamics and its control is an area of active current research, due to its application in supercontinuum generation [8].

By means of Fourier modes, the DCE of a square-integrable, bounded and mathematical function can be constructed [9,10]. In actual applications, it is regular situation that the square-integrable ceaseless function is the energy density $p(x)$ that is characterized on R and with Fourier change.

$$F(k) = \int \exp[-ix \cdot k]\rho(x)d^d x. \tag{1}$$

From now on, we consider just the case for $d = 1$. The modal fraction quantifies the relative weight of every mode which can be characterized as [9–11],

$$f(k) = \frac{|F(k)|^2}{\int |F(k)|^2 dk}. \tag{2}$$

The modal fraction normalized to ensure the positivity of DCE, is normalized by the mode carrying extreme weight $f_{max}(x)$,

$$\tilde{f}(k) = \frac{f(k)}{f_{\max}(k)}. \tag{3}$$

The numerical articulation of DCE is characterized as $S_c[\tilde{f}]$ is [9,10]

$$S_c[\tilde{f}] = -\int_{-\infty}^{\infty} \tilde{f}(k) \ln \tilde{f}(k) dk, \tag{4}$$

which speaks to an outright breaking point on the best lossless pressureof any communication [12]. In order to characterize DCE, one would utilize the Fourier series of the function $p(x)$for intermittent functions, while other conceivable useful changes could on a fundamental level be utilized to acquire the DCE. The away from translation of Fourier change relating expanded spatial restriction to more extensive energy mode dissemination, makes it the most proficient to characterize DCE, as the numerous applications referred to above have appeared.

In the present work, we have investigated DCE of for optical rogue wave in tapered graded-index waveguide. The non-linear Schrödinger equation possess all kind of non-linear excitations such as solitary wave, bright and dark soliton, and rouge waves as the solution. The one studied here is the GNLSE with solution of rogue wave. Since DCE forms a versatile framework to formalize uncertainty and predictability so our study extends its application to the non-linear world. The non-linear Schrödinger equation with solution of rogue wave is widely used in optical fiber communications, computer networks, long-distance telecommunications, and sensory receptor cells. It solves the problem of modal dispersion to a considerable extent. Since DCE forms a versatile framework to formalize uncertainty and predictability so our study extends its application to the non-linear world.

II Differential Configurational Entropy for Optical Rogue Wave in Tapered Graded-Index Waveguide

II.a Model equation and self-similar rogue wave solution

The beam propagation in tapered graded-index in nonlinear fiber amplifiers is modelled by the inhomogeneous nonlinear Schrödinger equation (NLSE) of the form

$$i\frac{\partial \Psi}{\partial z} + \frac{1}{2}a^2(z)\frac{\partial^2 \Psi}{\partial x^2} + \frac{1}{2}M(z)x\Psi - \frac{i}{2}g(z)\Psi + 2\gamma(z)\Psi + h(z)|\Psi|^2\Psi = 0, \tag{5}$$

where $+$ $(-)$ corresponds to the case of self-focusing (self-defocusing) and $a(z), M(z)$, and $\gamma(z)$ are the dispersion term, dimensionless profile function, and the external potential. The dimensionless variables z corresponds to propagation direction, x is the transverse componentand Ψ is the wave function.

Further, $g(z)$ is a dimensionless net gain (if $g > 0$)or loss of the energy (if $g < 0$) in the system. The general form of the dispersion can be written as

$$a(z) = \frac{d + \cos^2(z)}{\alpha}.$$

The self-similar optical rogue wave solution of Eq. (5) can be obtained by transforming it into a standard NLSE by using gauge and similarity transformations, together with generalized scaling of the z variable,

$$\Psi(x, z) = B(z)\Phi\left[\frac{x - x_c(z)}{\alpha(z)}, \zeta(z)\right]\exp[i\varphi(x, z)], \tag{6}$$

where $B(z), \alpha(z)$, and $x_c(z)$ are the dimensionless amplitude, width, and guiding-center coordinate of the beam, respectively. We assume a linear ansatz for the global phase:

$$\varphi(x, z) = [p_1(z)x + p_2(z)]. \tag{7}$$

substituting Eqs. (6) and (7) into Eq. (5), we obtain a set of first-order differential equations for the parameters of transformation Eqs. (6) such that the transformed field Φ satisfied the standard, homogeneous NLSE:

$$i\frac{\partial\Phi}{\partial\zeta} + \frac{1}{2}\frac{\partial^2\Phi}{\partial\chi^2} + h(z)|\Phi|^2\Phi = 0. \tag{8}$$

Here the effective propagation distance ζ is given by

$$\zeta(z) = \zeta_0 + \int_0^z B^2(z)dz \tag{9}$$

and we have introduce the similarity variable χ as

$$\chi(x, z) = \frac{[x - x_c(z)]}{\alpha(z)}, \tag{10}$$

The guiding-center position is given by the expression

$$x_c(z) = \alpha\left[\int_0^z \frac{a^2(z)p_1(z)dz}{k} + x_0\right] \tag{11}$$

The solution of Eqs. (5) can be obtained from the solution of NLSE using the self-similar transformation [2]. One of the acceptable solution is self-similar rogue wave solution whose mathematical expression is

$$\Psi(\chi, z) = \frac{(d + \cos^2(z))}{\alpha}\left[1 - \frac{4(1 + 2i\zeta)}{1 + 4\chi^2 + 4\zeta^2}\right]\exp[i\zeta]\exp[p_1(z)\alpha(\chi + x_c) + p_2(z)] \tag{12}$$

II.b Configuration entropy for nonlinear self-similar optical rogue wave

using normalization fractions, the position space energy density is calculated as,

$$\rho(x) = \frac{(1 - \frac{4(1-2\iota\zeta)}{1+4\zeta^2+4\chi^2})(1 - \frac{4(1+2\iota\zeta)}{1+4\zeta^2+4\chi^2})}{1 + \frac{8}{5+4\zeta^2}}, \tag{13}$$

By using the fourier transform. The momentum space information density for optical rogue wave is obtained of the corresponding position space energy density.

$$\rho(k) = \frac{a_1\sqrt{2\pi}Abs(k)}{a_2} + \frac{\sqrt{2\pi}DiracDelta(k)}{a_2} + \frac{2a_1\pi Abs(k)DiracDelta(k)}{(a_2)^2}, \tag{14}$$

$$a_1 = \exp\left[-\frac{Abs(k)}{2\sqrt{\frac{1}{1+4\zeta^2}}}\right],$$
$$a_2 = 1 + \frac{8}{5+4\zeta^2},$$

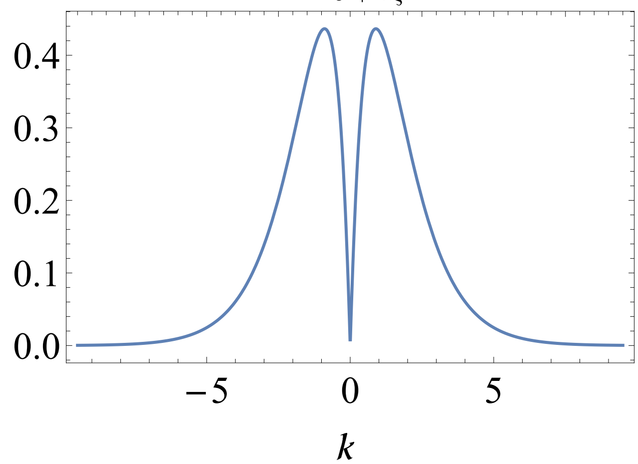

Figure 1: Modal fractions for rogue wave with $\alpha = 35.0$ and the maximum is at $k = 0$.

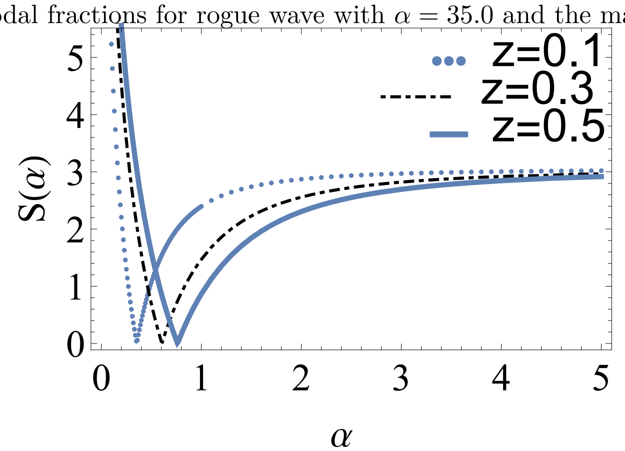

Figure 2: CE $S(\alpha)$ as a function of the rogue wave width α. The minimum value of CE occurs at different values of $\alpha = 0.35, 0.61, 0.76$.

The evaluation of DCE for the rogue wave as a function of the time parameter z was found utilizing for several values of width α. Alternatively, one may examine the dependence of $S(\alpha)$ on the width parameter α itself at different fixed times in the evolution of the rogue wave. The results are shown in Fig. 2 for three snapshots of $z = 0.1, 0.3, 0.5$. Overall, we find that $S(\alpha)$ is weakly dependent on z, with a global minimum at $\alpha \simeq 0.35, 0.65, 0.76$, respectively. As discussed in the literature ([9,10]), a minimum of DCE signals the most stable configuration with respect to a given parameter, in this case, the rogue wave width α. These values denote the best range for the rogue wave width to ensure its propagation through the tapered graded-index waveguide with optimal compression of information.

III Conclusion

This work establishes the utility of Differential Configurational Entropy (DCE) in characterizing the stability and information content of self-similar optical rogue waves within tapered graded-index waveguides. By analyzing the solution derived from the inhomogeneous nonlinear Schrödinger equation, we computed the DCE, $S(\alpha)$, as a function of the rogue wave width α at various propagation distances z. Our central finding is the identification of a well-defined global minimum in $S(\alpha)$ for a narrow range of α (approximately 0.35, 0.65, and 0.76 at $z = 0.1, 0.3, 0.5$, respectively). This minimum signifies the most stable configuration of the rogue wave, where the information required to describe its spatial profile is optimally compressed into its Fourier momentum modes. Physically, this corresponds to the optimal width at which the wave is most localized and its position is most predictable, a state of minimal dispersive spreading. Given the critical applications of tapered graded-index waveguides in optical fiber communications [13], long-distance telecommunications [14,15], and other photonic systems [16], our results provide a concrete, information-theoretic design principle. By tailoring system parameters to favor rogue wave formation near these low-entropy widths, one can enhance signal integrity and propagation efficiency.

In future work, this methodology can be extended to investigate the dependence of DCE on other rogue wave parameters and to more complex, higher-dimensional systems, further exploring the role of configurational entropy in nonlinear wave dynamics [10]. This approach paves the way for designing advanced waveguide systems with optimized performance for controlling and harnessing rogue waves.

Acknowledgment

The authors also acknowledge the Institute of Natural Science and Applied Technology, Kolkata for providing research facility.

References

[1] D. H. Peregrine. Water waves, nonlinear schrödinger equations and their solutions. *J. Aust. Math. Soc. Ser. B, Appl. Math.*, 25:16, 1983.

[2] N. Akhmediev, A. Ankiewicz, and M. Taki. Waves that appear from nowhere and disappear without a trace. *Phys. Lett. A*, 373:675, 2009.

[3] Y. V. Bludov, V. V. Konotop, and N. Akhmediev. Matter rogue waves. *Phys. Rev. A*, 80:033610, 2009.

[4] V. B. Efimov, A. N. Ganshin, G. V. Kolmakov, P. V. E. McClintock, and L. P. Mezhov-Deglin. Rogue waves in superfluid helium. *Eur. Phys. J. Spec. Top.*, 185:181, 2010.

[5] W. M. Moslem, P. K. Shukla, and B. Eliasson. Rogue waves in plasmas. *Europhys. Lett.*, 96:25002, 2011.

[6] M. Shats, H. Punzmann, and H. Xia. Capillary rogue waves. *Phys. Rev. Lett.*, 104:104503, 2010.

[7] D. R. Solli, C. Ropers, and B. Jalali. Active control of rogue waves for stimulated supercontinuum generation. *Phys. Rev. Lett.*, 101:233902, 2008.

[8] A. Ankiewicz, D. J. Kedziora, and N. Akhmediev. Rogue wave triplets. *Phys. Lett. A*, 375:2782, 2011.

[9] M. Gleiser and N. Stamatopoulos. Information content of spontaneous symmetry breaking. *Phys. Lett. B*, 713:304, 2012.

[10] M. Gleiser and D. Sowinski. Entropic measure of localized energy configurations: Kinks, bounces, and bubbles. *Phys. Rev. D*, 98:056026, 2018.

[11] C. E. Shannon and W. Weaver. *The Mathematical Theory of Communication*. University of Illinois Press, Urbana, 1949.

[12] M. Gleiser and N. Stamatopoulos. Information theoretic approach to the study of non-linear dynamics. *Phys. Rev. D*, 86:045004, 2012.

[13] J. Arrue, F. Jimenez, G. Aldabaldetreku, G. Durana, and J. Zubia. Design of tapered graded-index profile fibres for sensing applications. *J. Opt. Soc. Am.*, 16:16616, 2008.

[14] B. Sengupta, S. B. Laughlin, and J. E. Niven. Consequences of converting graded to action potentials upon neural information coding and energy efficiency. *PLoS Comput. Biol.*, 10(1):e1003439, 2014.

[15] P. Liu, B. Chen, and Z. W. Wang. Generation and manipulation of high-energy rogue waves in a superfluid. *Nat. Commun.*, 4:1, 2013.

[16] Q. Bi and K. Song. The rogue waves in the nonautonomous nonlinear systems. *J. Mod. Phys.*, 4:923, 2013.