Study of the fusion cross-section for some p-trapped reaction using single folding potential at low energy

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Abstract: The fusion cross-section plays a significant role in the synthesis of light elements in primordial nucleosynthesis as well as in the interior of compact stellar objects. We have studied the fusion cross-section of ${}^6{}_3$Li(p,γ)${}^7{}_4$Be, ${}^7{}_3$Li(p,γ)${}^4{}_2$He, ${}^7{}_4$Be(p,γ)${}^8{}_5$B and ${}^{10}{}_5$B(p,γ)${}^7{}_4$Be reactions by incorporating the single folded potential model. The Gaussian distribution has been used as an input for the matter density of the projectile and target particles in the single folding potential model. In this work, we theoretically investigated the energy dependence of the fusion cross-section of ${}^6{}_3$Li(p,γ)${}^7{}_4$Be, ${}^7{}_3$Li(p,α)${}^4{}_2$He, ${}^7{}_4$Be(p,γ)${}^8{}_5$B, and ${}^{10}{}_5$B(p,α)${}^7{}_4$Be reactions below the height of the Coulomb barrier. The fusion cross-section has been computed for the proton induced reactions in the sub-barrier energy regime (E $\sim$ 1 eV to few keV). The numerical computation of the observables is done in the frame work of the single fold potential model approach. The results of our calculation are compared with those found in the literature. The present results are in good agreement with the experimental results.

Keywords: Fusion Cross-section, M3Y-Potential, Primordial Nucleosynthesis

I Introduction:

Nuclear astrophysics is the union of nuclear physics and astrophysics, which explain the evolution of the stars. The nuclear reactions are responsible for the formation of all elements in the universe heavier than hydrogen and generating energy in stars [1,2]. The light-ion fusion reaction at low energy below the Coulomb barrier is an important physical phenomenon for the primordial nucleosynthesis in the stars [3,4].In 1939, H.A. Bethe explained the various nuclear reaction cycles for the stellar evolution with the emphasis of the production of light elements [5]. In 1957, Burbidge et al. [6] explained the mechanism of nucleosynthesis of elements in stars. In our literature review, we have found a large amount of data involving the nuclear reaction network and some of them are charged particles (protons and alpha) and some are charged particles (neutrons) [7,8]. Beyond the iron (Fe), the formation of elements has occurred through various processes like the r-process, s-process etc [9,10]. The charged particles contribute in the nuclear reaction network through transfer and capture processes. Comparing these two processes, the fusion cross section of the earlier is always greater than the later one [11,12]. For the difficulties of the direct measurement of the fusion cross section at astrophysically relevant energies, a smooth energy-dependent quantity astrophysical S-factor, S(E), has been introduced. The cross section is much lower than the height of the Coulomb barrier of the interaction nuclei in the low energy regime (E 1 eV to few keV) [13,14]. Theoretical modeling is highly needed for the explanation and justification of the experimental findings at astrophysical relevant energy.

In this work, for the computation of the cross section of light nuclei, we have used a dignified theoretical model that is a smooth function of energy. Here we adopt the single folding potential model for the computation of the astrophysical S-factor and fusion cross section for the light nuclei [15,16]. In 2000, Li et al. proposed the Selective Resonant Tunneling Model (SRTM) to compute the fusion cross section and compare the findings for the reaction D+T with the experimental results, which give a reasonable agreement [17]. In 2004, Li. et al. computed the fusion cross section for D+D and D+3He reactions [18]. In 2019, Singh et. al. [19] compute the fusion cross section by using the complex nuclear potential of

D+D, D+T, D+3He etc reactions. In this work, we have computed the fusion cross-sections for p+^{6}Li, p+^{7}Li, p+^{7}Be and p+^{10}B reactions at astrophysically relevant energy regime.

In Section II, we have introduced an overview of the theoretical framework of the fusion cross-section for some selected p-trapped nuclear reactions at low energy. Next, we have made comparisons with the available experimental results and corresponding theoretical results in Section III. In particular, we have compared the experimental results and theoretical results for the fusion cross-section at low energy, and the corresponding S-factor has been drawn. In the last Section (section IV), we close with a brief summary and conclusions.

II Theoretical Framework:

As a result of interactions between two nuclei, many nuclear reactions have occurred. The nucleus-nucleon interaction has been calculated within the framework of a single folding potential model in which the density of the nucleus has been folded in term of density [20]. The single folding potential model is a suitable method for the examination of the experimental findings. In this work, all the computations have been done based on the single folding potential model. Though the energy of the projectile nucleus is far below the height of the mutual Coulomb barrier, the fusion reaction takes place due to the quantum tunneling phenomenon. The fusion cross-section of the reaction(below the Coulomb barrier height) can be expressed as an energy dependent quantity and is given in equation 1.

$$\sigma(E) = \frac{S(E)}{E} exp(-\int_{r1}^{r2} \sqrt{\frac{2\mu(Veff - E)}{\hbar^2}} dr) \tag{1}$$

Here μ is the reduced mass of the nuclear system, and $\hbar$ is the reduced planck's constant and E is the projectile nucleus energy. The astrophysical S(E), from ref. [21], is given in equation 2.

$$S(E) = S_0 + S_1.E + S_2.E^2 \tag{2}$$

Where S_0, S_1 , S_2 are the constants. The remaining term in equation-1 i.e. V_{eff} is given in equation 3 [22].

$$V_{eff}(r) = V_C(r) + V_N(r) + V_{Centi}(r) \tag{3}$$

Here, in equation 3,the first term denotes the Coulomb potential. The effective potential dependence of the charge of the projectile and target of a colliding nuclear system [23]. The Coulomb potential has been given in equation 4.

$$V_C(r) = \begin{cases} 1.44\frac{Z_1Z_2}{r}; \text{for r} > \text{R [MeV]} \\ 1.44\frac{Z_1Z_2}{2R}(3 - \frac{r^2}{R^2}); \text{for r} \leq \text{R [MeV]} \end{cases} \tag{4}$$

Here, Z_1 and Z_2 are the charges of the nuclear system, and R is the radius of the sphere of the target nuclei. The last term of equation3 is the centrifugal barrier of the nuclear system and has been given in equation 5.

$$V_{Centi}(r) = \frac{\hbar^2 l(l+1)}{2\mu r^2} \tag{5}$$

Here, l is the angular momentum, and r is the radius of the target nucleus. The remaining term of the effective potential is the complex nuclear potential function *VN (r)* described as single folding potential *VSF (r)*. The single folding potential [24] model furnishes precious information about the nuclear system. The fundamental inputs in the single folding model potential computation are the nuclear densities of the colliding nuclear system.

The single folding model potential is density dependent. The matter density distribution of nuclear systems has many natures such as Fermi distribution, Gaussian distribution, and Variational Monte Carlo (VMC) [11]. In single folding model calculation, we have use the Gaussian matter distribution which is exponentially decreasing in nature. From various kinds of effective interaction, we use DDM3Y-type interaction for the calculation of double folding model potential. The DDM3Y-type interaction acts on a short range of the nuclear density and contains no explicit density dependence. The density distribution used in the single folding potential calculations is very important in examining nuclear reactions. The single folding potential of the nuclear system has been represented by equation 6.

$$V_{SF}(r) = \int_V \rho(r) V_{NN}(s) d^3r \tag{6}$$

Where r is the distance between the projectile and target nuclei.

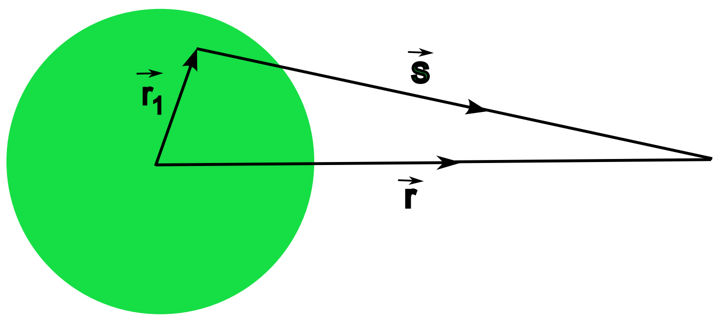

Figure 1: Coordinates used in single-folding potential calculations For the nuclei of projectile and target

The nucleon-nucleon distance(s) can be written in terms of r, r_1, r_2 and represented by equation 7.

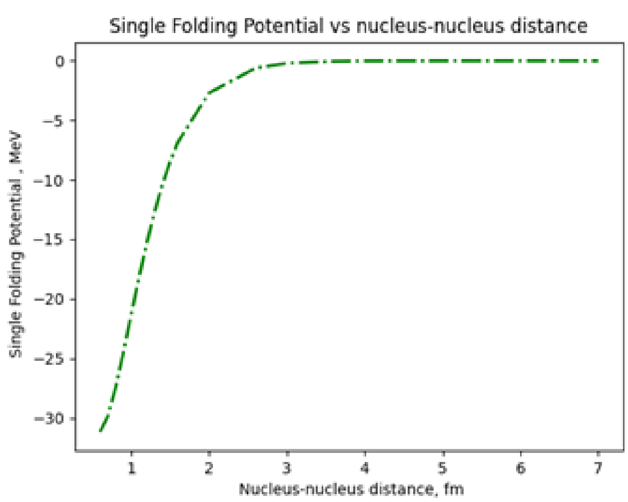

Figure 2: The shape of double-folding potential for the nuclei projectile and target

$$\vec{s} = \vec{r} - \vec{r_1} \tag{7}$$

The term in equation 6 denotes the nucleon-nucleon interaction. The Fermi and Gauss distributions of nuclear are given by equation 8 and equation 9.

$$\rho(r) = \frac{\rho_o}{[1 + exp(r - c)/a]} \tag{8}$$

Where ρ_o = 2.607 (fm^3), c = 2.0(fm), a= 0.486(fm)

$$\rho(r) = c\exp(-(r/\alpha)^2) \tag{9}$$

Where c = 2.0 (fm-3), α = 2.08207 (fm)

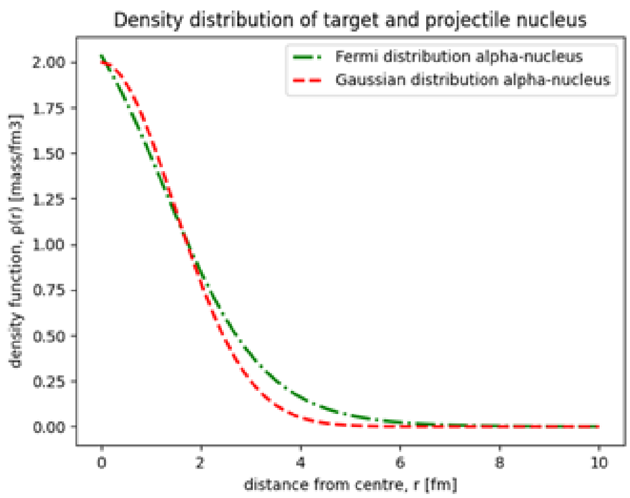

Figure 3: Nuclear matter distribution of reaction projectile+ target

There are two types of DDM3Y-type nucleon-nucleon interactions [28], the DDM3Y-Reid nucleon-nucleon interaction and the DDM3Y-Paris effective nucleon-nucleon interaction, and which can be represented by the equation 10 and equation 11.

$$DDM3Y - Reid : v(r) = 7999(exp(-4r)/4r)\breve{}2134(exp(-2.5r)/2.5r) \tag{10}$$

$$DDM3Y - Paris : v(r) = 11062(exp(-4r)/4r)\breve{}2538(exp(-2.5r)/2.5r) \tag{11}$$

The Schrodinger equation for the effective potential Veff is given in equation 12.

$$(\nabla^2 + \frac{2\mu V_{eff}}{\hbar^2} - k^2)\psi(r) = 0 \tag{12}$$

Where k^2 = $\frac{2\mu E}{\hbar^2}$, and ψ(r) appear for the sum of the nuclear and Coulomb wave functions and have been given in equation 13.

$$\psi(r) = \psi_N(r) + \psi_C(r) \tag{13}$$

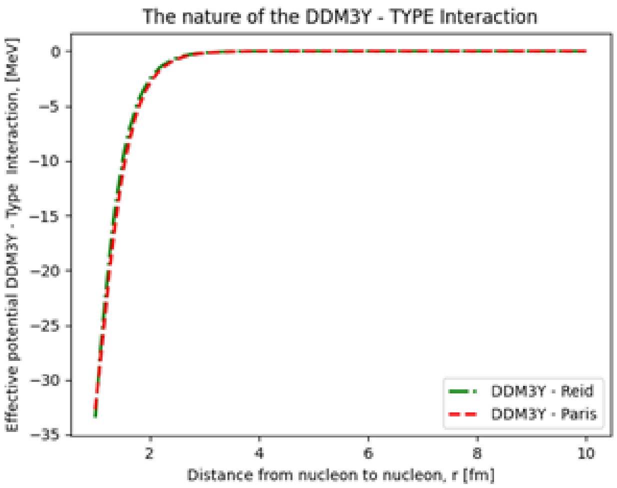

Figure 4: the shape of the DDM3Y-Type interaction for the reaction projectile + target

In the equation 13, the second term i.e. the Coulomb wave function holds only the incoming wave. The first term of equation 13 of the right hand appears for the outgoing nuclear wave. Equation 13 has been reduced to one-dimension r dependent only and has been given in equation 14.

$$(-\frac{d^2}{dr^2} + \frac{l(l+1)}{r^2} + \frac{2\mu V_{eff}}{\hbar^2} - k^2)\psi(r) = 0 \tag{14}$$

Where 'l" is the angular momentum quantum number. Now, when a proton nucleus is introduced into another nucleus for the nuclear reaction, the relative motion of that nuclear system can be described in terms of the wave function ψ(r) of that system which is a space dependent quantity, and the general solution ψ(r, t) of Schrodinger equation 15 for the interacting nuclei has been given in equation 15.

$$\psi(r,t) = \frac{1}{\sqrt{4\pi r)}}\psi(r)exp(-iEt/\hbar^2) \tag{15}$$

III Results and discussion:

The fusion cross sections of protons for the reactions ${}^6{}_3$Li(p,γ)${}^7{}_4$Be, ${}^7{}_3$Li(p,γ)${}^4{}_2$He, ${}^7{}_4$Be(p,γ)${}^8{}_5$B and ${}^{10}{}_5$B(p,γ)${}^7{}_4$Be have been measured at astrophysically relevant energies (E $\sim$ 1 eV to few keV). We have studied the fusion cross section for these four reactions within the framework of a single folding potential model below the height of the Coulomb barrier. The Gaussian shape of the matter distribution has been used in the single folding model. In single folding model, the free parameter is the normalization constant which is taken as unity in whole process of the computation. We have computed the fusion cross section by fitting the parametrization equation of the astrophysical S-factor. For the computation of the fusion cross section of the ${}^6{}_3$Li(p,γ)${}^7{}_4$Be reaction, we have taken S_0 = 1.2 keV mb, S_1 = 1.2x10^{-2} mb and S2 = 1.1x10^{-4} mb/keV for the calculation of the astrophysical S-factor and the result are displayed in figure -5. The Top panel is the fitting of astrophysical S-factor, and the Bottom panel is the comparison between the experimental results [25] and the computed results. The plot in the Bottom panel of the figure 5 shows a good agreement. For reaction ${}^7{}_3$Li(p,γ)${}^4{}_2$He, we take the parameters S_0 = 3.2 keV mb, S_1 = 3.2x10^{-1} mb, and S_2 = 1.55x10^{-1} mb/keV, and the result has been displayed in figure 6. The results show a good agreement between the experimental result [26], and the computed results. For reaction ${}^7{}_4$Be(p,γ)${}^8{}_5$B, we take the parameters S_0 = 0.0021 keV bm, S_1 = 0.031x10^{-2}

Table 1: **Parameters for the calculation of astrophysical S-factor**

Reactions	S_0 (Kev mb)	S_1 (mb)	S_2 (mb/KeV)
${}^6{}_3Li(p,\gamma)^7{}_4Be$	1.2	1.2x10^{-2}	1.1x10^{-4}
${}^7{}_3Li(p,\gamma)^4{}_2He$	3.2	3.2x10^{-1}	1.55x10^{-1}
${}^7{}_4Be(p,\gamma)^8{}_5B$	0.0021	0.031x10^{-2}	0.03x10^{-4}
${}^{10}{}_5B(p,\gamma)^7{}_4Be$	0.05	0.05	0.067

mb, and S_2 = 0.03x10^{-4} mb/keV and the result has been displayed in figure 6. The results show a good agreement between the experimental result [27] and the computed results. At last, for reaction ${}^{10}{}_5B(p,\alpha)^7{}_4Be$, we take the parameters S_0 = 0.05 keV mb, S_1 = 0.05 mb, and S_2 = 0.067 mb/keV and the result is displayed in figure 6. The results show a good agreement between the experimental result [28] and the computed results.

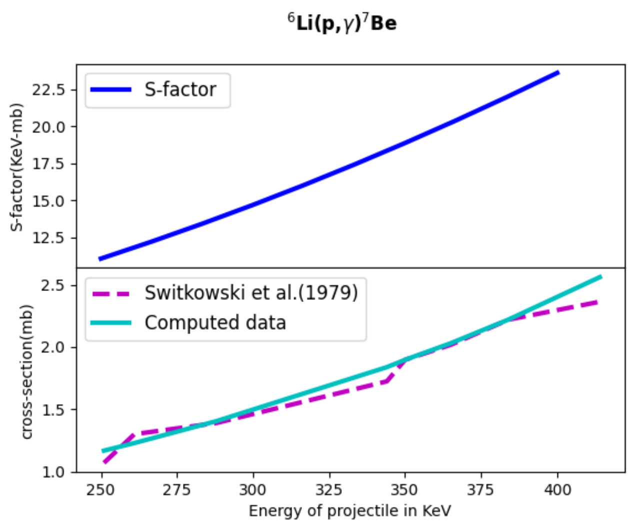

Figure 5: Top panel-Fitting of Astrophysical S-factor by using equation 2 and table 1. Astrophysical S-factor is in KeV mb and energy in KeV in the lab system. Bottom panel- Comparison between experimental data points and computed data points for ${}^6{}_3Li(p,\gamma)^7{}_4Be$ reaction. The experimental data points are taken from [25]. Cross section is in μb and energy in KeV in the lab system.

IV Summary and conclusion:

The astrophysical S-factor is broadly used in nuclear astrophysics for the extrapolations of the fusion cross section relevant to the Gamow energy regime. We have investigated ${}^6{}_3Li(p,\gamma)^7{}_4Be$, ${}^7{}_3Li(p,\gamma)^4{}_2He$, ${}^7{}_4Be(p,\gamma)^8{}_5B$ and ${}^{10}{}_5B(p,\alpha)^7{}_4Be$ reactions are far below the height of the Coulomb barrier. Although there are many models to analyze the fusion cross section, single folding potential model is a popular procedure for analyzing the experimental findings. We can see that the fusion cross section gives a good agreement between experimental results, and computed results and all the results have been displayed in figures-5-8. All the stellar nuclear reactions have occurred at astrophysical relevant energies, and at low energy, the measurement of astrophysical quantities is very hard. For a better theoretical approach, we have incorporated the single fold potential model below the height of the Coulomb barrier. So, to fit the experimental data with the computed data, theoretical modeling is highly needed with greater accuracy. The theoretical calculation can be done repeatedly.

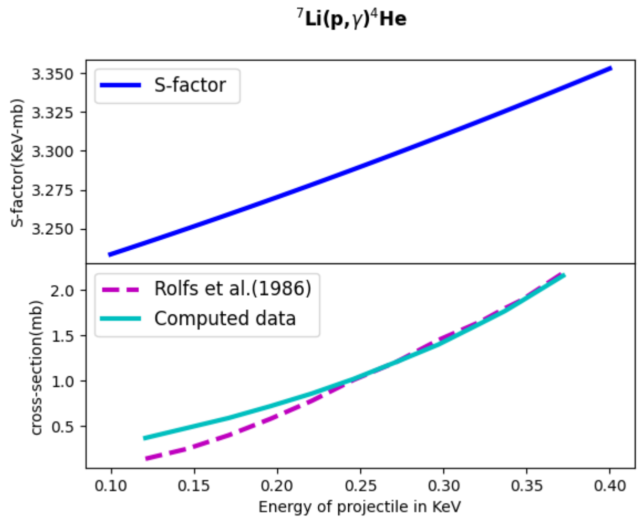

Figure 6: Top panel-Fitting of Astrophysical S-factor by using equation 2 and table 1. Astrophysical S-factor is in KeV mb and energy in KeV in the lab system. Bottom panel- Comparison between experimental data points and computed data points for ${}^{7}{}_{3}Li(p,\gamma){}^{4}{}_{2}He$ reaction. The experimental data points are taken from [26]. Cross section is in mb and energy in KeV in the lab system.

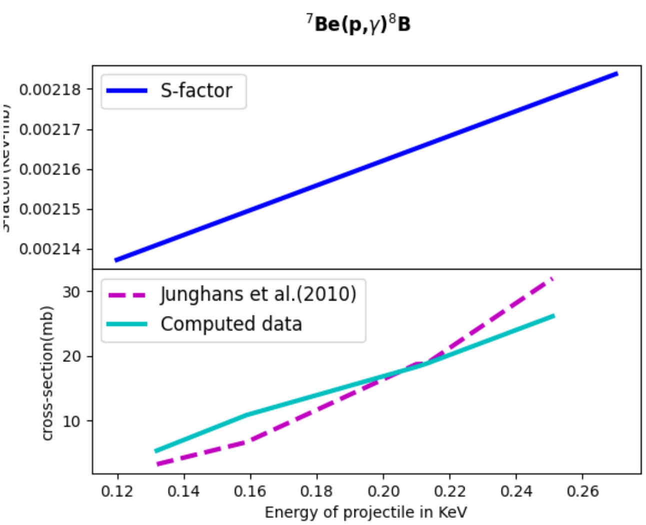

Figure 7: Top panel-Fitting of Astrophysical S-factor by using equation 2 and table 1. Astrophysical S-factor is in KeV mb and energy in KeV in the lab system. Bottom panel- Comparison between experimental data points and computed data points for ${}^{7}{}_{4}Be(p,\gamma){}^{8}{}_{5}B$ reaction. The experimental data points are taken from [27]. Cross section is in nb and energy in KeV in the lab system.

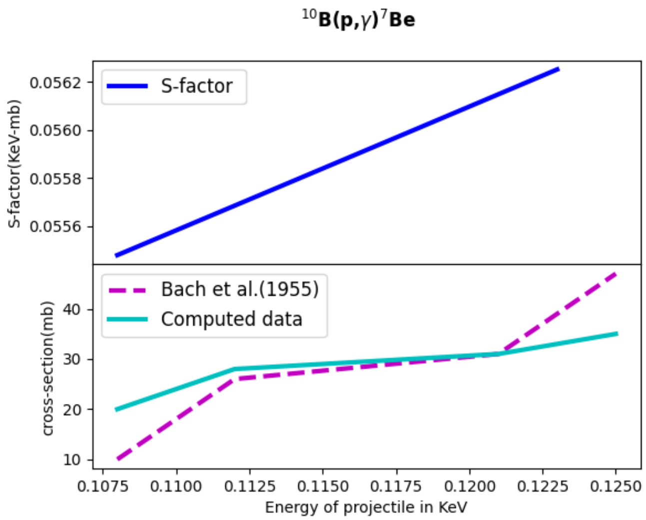

Figure 8: Top panel-Fitting of Astrophysical S-factor by using equation-2 and table-1. Astrophysical S-factor is in KeV mb and energy in KeV in the lab system. Bottom panel- Comparison between experimental data points and computed data points for ${}^{10}{}_{5}B(p,\alpha){}^{7}{}_{4}Be$ reaction. The experimental data points are taken from [28]. Cross section is in μb and energy in KeV in the lab system.

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