Laser-Driven Acoustic Instabilities In Materials With Strain-Dependent Dielectric Constants In Quantum Plasma

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Abstract: An analytical analysis of the laser-driven acoustic wave instability in a material with strain-dependent dielectric constants is given. The analysis is based on the quantum hydrodynamic model of a plasma-dominated regime. Using coupled mode theory, the acoustic instability in the medium is investigated. It is found large values of growth rates can be achieved for material having an anomalously large dielectric constant, using ferroelectric material. In our study we have used PZT and $BaTiO_3$ for further calculation. We have shown a comparative analysis for classical and quantum effect for both materials. This study investigates laser-driven acoustic instabilities in materials characterized by strain-dependent dielectric constants within the framework of quantum plasma physics. By considering the impact of both intense laser fields and material strain on the dielectric properties of the medium, we explore the resulting nonlinear interactions that give rise to acoustic wave instabilities. The strain-induced variations in the dielectric constant alter the plasma's response to external perturbations, leading to complex dynamic behaviour in the system. Utilizing a quantum hydrodynamic model, we derive the governing equations that describe these instabilities and examine their dependence on key parameters such as laser intensity, material strain, and quantum effects. The analysis reveals that the strain dependence of the dielectric constant significantly influences the threshold conditions for instability, as well as the growth rate and frequency characteristics of the acoustic waves. This research has important implications for the understanding and control of laser-material interactions in advanced plasma technologies, particularly in the context of quantum plasmas and high-intensity laser applications.

Keywords: Lasers, acoustic instability, QHD model, dielectric constants, ferroelectric materials.

I Introduction

When a strong laser beam interacts with a plasma, it can produce acoustic waves, or sound waves, that can become unstable and increase in amplitude. This instability can disrupt the stability of the plasma and have an impact on processes like laser fusion or particle acceleration. The ponderomotive force that the laser exerts on the plasma electrons causes density perturbations that propagate as acoustic waves. This phenomenon is known as laser driven acoustic instabilities in plasma. A localized density modulation that can serve as an acoustic wave source is produced when a high-intensity laser beam interacts with a plasma. This occurs when the oscillating electric field of the light drives electrons away from the centre of the laser beam. Stronger ponderomotive forces from higher laser intensities encourage the expansion of acoustic waves. The instability development is influenced by the plasma density, which also impacts the acoustic waves' dampening and speed of propagation. The kinetics of the interaction and the rate at which the instability grows can be affected by the laser pulse's length. Classically according to the description above, a crystal with SDDC can readily attain an abnormally high growth rate for amplification of acoustic waves caused by laser beam irradiation, but it will require a significantly greater value of high frequency oscillatory electric field than crystals with piezoelectric connection. This implies that greater acoustic power could be transported by SDDC crystals than by piezoelectric crystals. It is evident from the study that spatial non-uniformity has little bearing on non-linear interactions. The study of non-linear interactions in crystals with Strain Dependent Dielectric Constant (SDDC) may thus be a valuable technique, with possible applications in solid-state diagnostics and energy conversion in crystals with high dielectric constants [1–3]. In a piezoelectric crystal, an electric field often travels with an acoustic wave. The electric field creates currents and space charge when the crystal is also semiconducting, which causes dispersion and acoustic loss [4,5].

Many studies on the interaction of ultrasonics with common materials have been published in the last few decades with the goal of testing materials without causing any damage [6]. The need to comprehend how sound travels through piezoelectric crystals has been recognized by several research teams worldwide due to the development of actuators and transducers. However, the research of surface wave propagation received the most interest in the case of piezoelectric crystals [7]. The propagation of sound in biased piezoelectric crystals with any type of symmetry is described theoretically. Due to bias pressure, the case of lithium niobate is emphasized. For each direction, the change in slowness (inverse velocity) due to this pressure is computed. Additionally, the impact of stress on energy flow and acoustic polarization is described. The description of shifting slowness surfaces is not restricted to homogeneous plane waves due to a bias field; inhomogeneous plane waves are also considered [8].

According to the analysis and comparison in this paper, ferroelectric semiconductors grew at a faster rate than piezoelectric semiconductors. Plotting the electric field against wave number shows that the waves are linear in quantum effect and nonlinear in nonquantum effect. It also shows how the electric field varies with number density, steady state gain with carrier density, and steady state gain with wave number. The ferroelectric semiconductor employed in this work grew at a faster rate than the piezoelectric semiconductor [9].

The amplification happens at relatively low values of the dc electric field as the temperature gradient increases. When a temperature gradient is introduced, the attenuation of an acoustic wave transitions to its amplification at the drift field where the drift velocity of electrons is less than the velocity of sound for the majority of materials with strain-dependent dielectric constants [10]. This substance is a member of the piezoelectric material class. Electrical voltage is often produced across the boundaries of a piezoelectric material system when it is subjected to external mechanical action. Thus, when a PZT material is subjected to mechanical pressure, it produces electrical voltage that can be utilized in capacitors, sensors, batteries, memory, and other devices [11–13].

PZT has its remarkable electrical characteristics, which include large d_{33} (the piezoelectric charge coefficient) and g_{33} (the piezoelectric voltage coefficient), high Curie temperature.

The third order susceptibility and nonlinear current density are examined in various wave number areas with and without the quantum effect. The threshold pump intensity changes qualitatively with wave number both with and without the quantum effect. It is discovered that the threshold characteristics are altered by the quantum correction via Fermi temperature and Bohm potential parameters, large dielectric constant [14,15], low tangent loss and more [12]. PZT has developed into a practical material system for capturing the energy of human vibrations due to its extraordinary piezoelectric qualities [16].

A detailed investigation of quantum and relativistic corrections to KdV and envelope solitons in Ion-Plasma Waves, which can offer valuable parallels to the modulational behaviour, described by Sahoo et. al [17]. Similarly, Shilpi et al. [18] discuss quantum-electron acoustic solitary structures in multi-temperature Fermi plasmas, presenting a complementary perspective on nonlinear wave formation relevant to the present model. The Semi-Lagrangian framework employed by Das et al. [19] offers an efficient alternative methodology for studying nonlinear electrostatic waves in quantum plasma. Furthermore, Mahanta et al. [20] examine impact of ion pressure anisotropy effects in collisional quantum magneto-plasmas with heavy and light ions, where anisotropic pressure terms become significant.

II Theoretical Formulations

We have used a quantum hydrodynamical model of a homogeneous one component (electron only) material of infinites extent, in which the only coupling between conduction electrons and acoustics waves is due to the SDDC. The dielectric constant is given by,

$$\varepsilon = \varepsilon_0(1 + gS) \tag{1}$$

where ε_0 is the dielectric constant in absence of strain S and g is the coupling constant. Hence the electric displacement is given by

$$\overrightarrow{D} = \epsilon_0 \overrightarrow{E} + \epsilon_0 g S \overrightarrow{E} \tag{2}$$

The thermodynamic requirement $\left(\frac{\partial D}{\partial S}\right)_E = -\left(\frac{\partial T}{\partial E}\right)_S$ provides a straightforward explanation of the constituent interactions affecting the material's stress. Consequently, the material's stress is as follows:

$$\overrightarrow{T} = CS - \epsilon_0 g E_0 \overrightarrow{E} \tag{3}$$

The first term is usual Hook's law contribution and C is the elastic stiffness constant.

Assuming that $\omega_0\,(\approx \omega_P) \gg v$, a high frequency laser beam, $E_0 \exp\left[i\left(k_0 x - \omega_0 t\right)\right]$, is applied parallel to the direction of wave propagation (along the x axis). The following are the other fundamental equations that were employed in this investigation,

$$\frac{\partial v_0}{\partial t} = \left(\frac{e}{m}\right) E_0 = \upsilon v_0 \tag{4}$$

$$\frac{\partial E}{\partial x} = \frac{en}{\epsilon_0} - \left(\frac{\epsilon_0 g E_0}{\epsilon_0}\right) \frac{\partial^2 U}{\partial x^2} \tag{5}$$

$$\frac{\partial v_1}{\partial t} + \left(v_0 \cdot \nabla\right) v_1 + v_1 \upsilon = \left(\frac{e}{m}\right) E_1 - \frac{1}{mn} \nabla P + \frac{\hbar^2}{4m^2 n_0} \nabla^3 \tag{6}$$

$$\frac{\partial n_1}{\partial t} = v_0 \frac{\partial n_1}{\partial t} + n_0 \frac{\partial v_1}{\partial t} \tag{7}$$

$$\rho \frac{\partial^2 U}{\partial t^2} - 2\rho \chi_e \frac{\partial U}{\partial t} + \left(\frac{\epsilon_0 g E_0}{\epsilon_0}\right) \frac{\partial E}{\partial x} = C \frac{\partial^2 U}{\partial x^2} \tag{8}$$

The zeroth-order equation of motion for electrons, represented by equation (4), indicates that the oscillating electric field, $E_0 \exp\left[i\left(k_0 x - \omega_0 t\right)\right]$, will cause the electrons to oscillate. The velocity of the electrons is denoted by v_0.

Poisson's equation [Equation (5)], which gives the SDDC contribution to the polarization on the right-hand side, was used to derive the space charge field, E. The perturbed and equilibrium electron densities are denoted by n_1 and n_0, respectively.

Equation (6) is the first order momentum equation and (7) is the first order continuity equation for electrons, where k_B is the Boltzmann's constant, T is the electron temperature, v is the first-order perturbed velocity, and e is the charge of the electrons.

Equation (8) describe the motion of the lattice in a crystal, specifically in the context of elasticity theory, possibly for a piezoelectric or electromechanically coupled material such as an SDDC crystal. The terms involve the dynamics of lattice displacements under mechanical and electric influences, with damping effects and wave propagation properties. Where U is the lattice displacement and ρ is the mass density of the crystal.

The acoustic wave has a wave vector k and an angular frequency ω, so that the low frequency perturbations are proportional to $\exp\left[i\left(k_0 x - \omega_0 t\right)\right]$ and $\omega \ll \omega_0$. A longitudinal electric field is produced because of acoustic wave is propagating along a specific direction in the crystal. Using equation (5) and (8),

$$\left(\omega^2 - v_s^2 k^2 - \frac{g^2 {E_0}^2 k^2}{\rho}\right) U = \frac{2i\rho\chi_e \omega U}{\rho} - \frac{gE_0 en}{\rho E_0} \tag{9}$$

And using equation (4) and (6), in collision dominated regime, we obtain,

$$\frac{\partial^2 U}{\partial t^2} + v\frac{\partial n}{\partial t} + \omega_p^{'2} + \left(en_0 \frac{(\epsilon_0 g E_0)}{m{\epsilon_0}^2}\right)\frac{\partial^2 U}{\partial x^2} = -iknE \tag{10}$$

where in the above equation p is the pressure, $p = \frac{mv_F n_1^3}{3n_0}$, $\omega_p^{'2} = \omega_p^2 + k^2 v_F$, $E = -\frac{e}{m}E_0$, $v'_F = v_F\sqrt{1+\gamma_e}$, $v_F = \frac{2k_B T_F}{m}$ is Fermi Speed, k_B is Boltzmann Constant, T_F Fermi temperature of electron. $\gamma_e = \frac{\hbar^2 k^2}{\delta m k_B T_F}$, $\omega_p = \sqrt{\frac{n_0 e^2}{m\epsilon_0}}$.

The density perturbation n in the plasma is considered for any $E_0 \exp\left[i\left(k_0 x - \omega_0 t\right)\right]$, and density perturbation is caused by force wave disturbance at $(\omega_0 + \omega)$ the upper and $(\omega_0 - \omega)$ lower side band frequencies. $\omega = \omega_1 + \omega_0$ and $k = k_1 + k_0$ known as the momentum and energy conservation relations. By these side-bands equation (10) yields the following expressions,

$$n(\omega_+ k_+) = \frac{ik^3\beta^2 n_0 \epsilon_0 geE}{m\epsilon\rho\left(\omega^2 - k^2v^2 + 2i\chi_e\omega\right)\left(-\omega_+^2 - i\omega_+\nu + \omega_p'^2 + ik_+E\right)} \tag{11}$$

$$n(\omega_- k_-) = \frac{ik^3\beta^2 n_0 \epsilon_0 geE}{m\epsilon\rho\left(\omega^2 - k^2v^2 + 2i\chi_e\omega\right)\left(-\omega_-^2 - i\omega_-\nu + \omega_p'^2 + ik_-E\right)} \tag{12}$$

Since the total effective polarization are of upper and lower band frequencies can be expressed as follow,

$$P_{eff} = \frac{\omega_p^{'2}\omega_p^2\epsilon\epsilon_0 gAk^2E_0E\left(\delta^2+v^2\right)}{m^2\left(\omega_s^2 + k^2v_s^2 + 2i\chi_e\omega_s\right)\left(\omega_0^2 - k^2v_0^2\right)\left[\left(\delta^2 + v^2 - \frac{k^2E^2}{\omega_0^2}\right) + \frac{4k^2\delta^2E^2}{\omega_0^2}\right]} \tag{13}$$

The induced polarization due to cubic nonlinearities is defined as,

$$P_{eff} = \epsilon_0 \chi_{eff}^{(3)} \left|E_0\right|^2 E \tag{14}$$

From equations (12) and (13), this leads to the third order nonlinear susceptibility including Quantum Mechanical effect as,

$$\chi_{eff}^{(3)} = \frac{\omega_p^{'2}\omega_p^2\epsilon\epsilon_0 gAk^2\left(\delta^2+v^2\right)}{m^2\left(\omega_s^2 + k^2v_s^2 + 2i\chi_e\omega_s\right)\left(\omega_0^2 - k^2v_0^2\right)}\left[\left(\delta^2 + v^2 - \frac{k^2E^2}{\omega_0^2}\right) + \frac{4k^2\delta^2E^2}{\omega_0^2}\right]^{-1} \tag{15}$$

In order to find the possibility of amplification in a semiconductor, we employ the relation,

$$\alpha_{eff} = \frac{k}{2\epsilon_0}\chi_{eff}^{(3)}\frac{\epsilon_0 g}{E_0 l^2} \tag{16}$$

Thus the growth rate of the amplified acoustic beam

$$G = \frac{\omega_p^{'2}\omega_p^2\epsilon\epsilon_0 gAk^2\left(\delta^2+v^2\right)\left(\omega_s^2 - k^2v_s^2\right)}{m^2 2\epsilon_1\left(\left(\omega_s^2 - k^2v_s^2\right)^2 + 4\alpha_e^2\omega_s^2\right)\left(\omega_0^2 - k^2v_0^2\right)\left[\left(\delta^2 + v^2 - \frac{k^2E^2}{\omega_0^2}\right) + \frac{4k^2\delta^2E^2}{\omega_0^2}\right]^{-1}} \tag{17}$$

Thus the instability threshold value of high frequency oscillatory electric field E_{0th}, is obtained as,

$$E_{0th} = \frac{m}{2ek}\left(\omega_0^2 - k^2 v_0^2\right)\epsilon_0 g\sqrt{\delta^2 + v^2} \tag{18}$$

$$E_{0th} = \frac{G^2\left(\delta^2 + v^2\right)}{\left(k^2\chi_{eff}^{(3)}\right)^2} \tag{19}$$

III Results and Discussion

An analytical investigation of the quantum effect of the laser driven instability of a laser beam and the consequent amplification of the shear acoustic wave in SDDC ferroelectric materials has been outlined here.

Table 1: Material parameters for PZT and $BaTiO_3$.

Parameters	**PZT**	**$BaTiO_3$**
m	$0.009m_0$	$0.0145m_0$
δ	10^6 S^{-1}	10^9–10^{11} S^{-1}
v	10^7 S^{-1}	10^9 S^{-1}
χ	10^{-12} m^2/V^2	10^{-14}–10^{-13} m^2/V^2
ρ	7.45×10^3 kgm^{-3}	6.02×10^3 kgm^{-3}

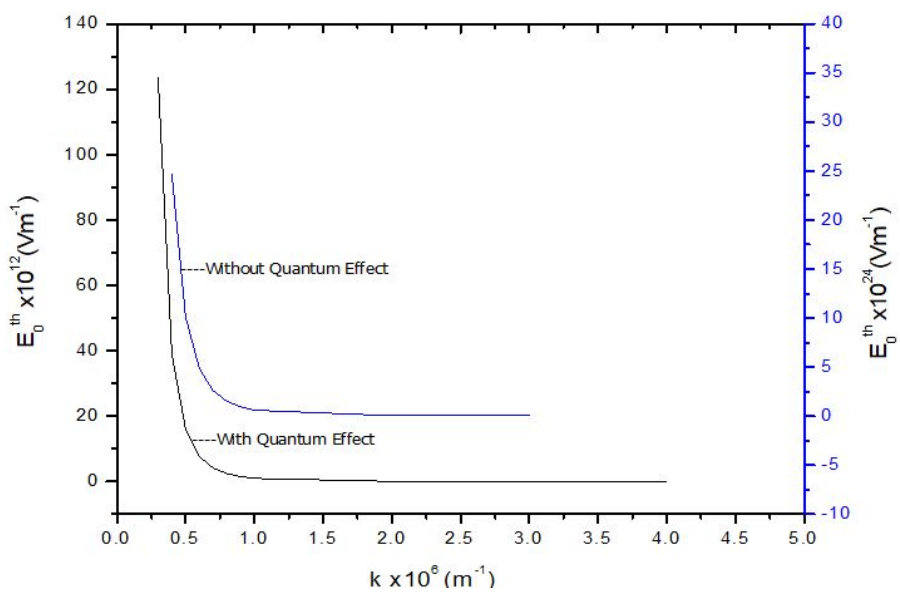

Figure 1: Threshold electric field vs. wave number for PZT with and without quantum effect.

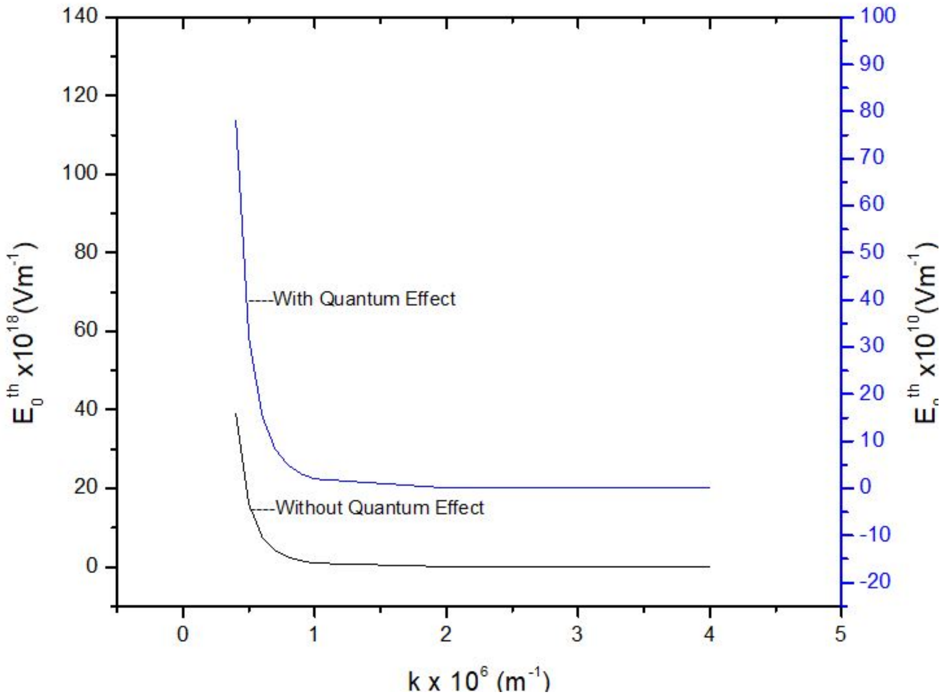

Figure 2: Variation of Threshold electric field vs. wave number for $BaTiO_3$ with and without quantum effect.

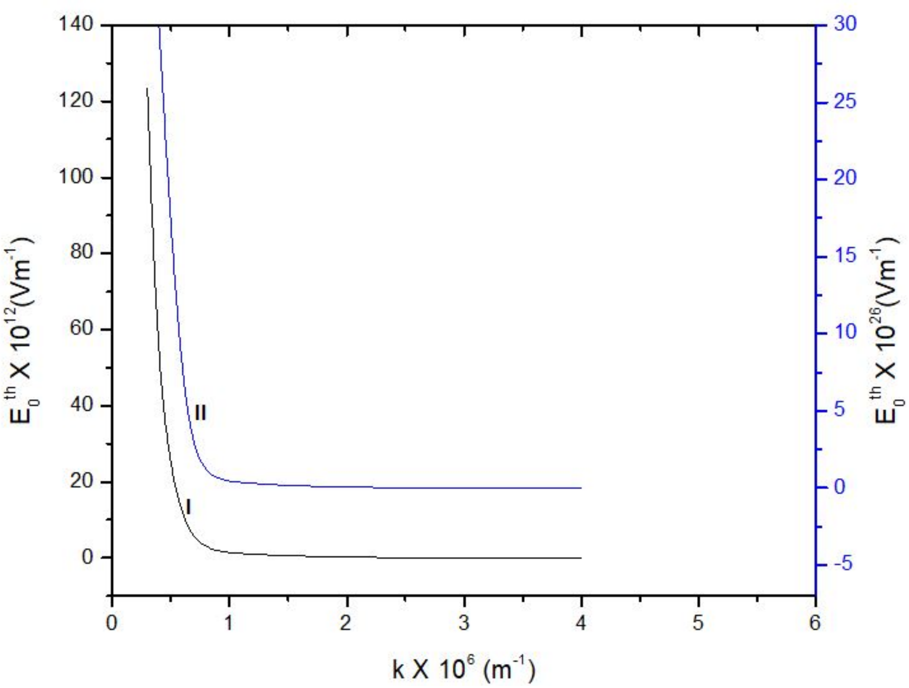

Figure 3: Quantum effect variation of the threshold electric field E_{0th} with wave number k in PZT(I) and $BaTiO_3$(II).

The threshold characteristics are illustrated in Figure 1 and Figure 2 with and without quantum effect using materials parameters. As usual, the threshold E_{0th} decreases as the wave number k increases for both the curve. It is found that for lower values of k, E_{0th} sharply decreases, while E_{0th} decreases with lower decapitation rate in case of the higher values of wave vector k. Threshold value is also found to be influenced by quantum effect through δ. In high doping regime, plasma mode dispersion dominates of quantum effect increases the value of E_{0th}. Figure 3 shows the quantum effect variation of the threshold electric field E_{0th}, with wave number k in PZT and $BaTiO_3$.

IV Conclusion

In this work, we have conducted an analytical investigation into the laser-driven acoustic instabilities in materials possessing strain-dependent dielectric constants, within the framework of quantum plasma physics. By employing a quantum hydrodynamic model and coupled-mode theory, we derived the governing equations for the instability and examined the influence of key parameters such as laser intensity, material strain, quantum corrections, and wave number on the growth rate and threshold conditions for acoustic wave amplification.

Our analysis reveals that materials with anomalously large dielectric constants, such as ferroelectric semiconductors PZT and $BaTiO_3$, exhibit significantly enhanced growth rates for laser-driven acoustic instabilities compared to classical piezoelectric materials. The strain dependence of the dielectric constant introduces a strong nonlinear coupling between the laser field and acoustic modes, lowering the threshold for instability and facilitating more efficient energy transfer.

The incorporation of quantum mechanical effects—through the Fermi temperature and Bohm potential—substantially modifies the instability characteristics. Quantum corrections were found to increase the threshold electric field E_0^{th} in high-doping regimes, while also influencing the dispersion and damping of acoustic waves. Comparative studies between classical and quantum treatments highlight the importance of quantum effects in accurately modeling high-density or low-temperature plasmas.

The results presented here have significant implications for advanced plasma technologies, including laser fusion, particle acceleration, and solid-state energy conversion. The enhanced acoustic gain in strain-sensitive ferroelectrics suggests promising applications in acoustic amplification, sensing, and vibration energy harvesting. Furthermore, the theoretical framework developed can be extended to other quantum-plasma systems and nonlinear wave phenomena.

Future work may explore experimental validation of these predictions, extension to multi-component plasmas, inclusion of relativistic effects, and investigation of nonlinear wave structures such as solitons and envelope modes in SDDC materials under high-intensity laser irradiation.

References

[1] S. Ghosh and S. Dixit. Laser driven acoustic instabilities in materials with strain-dependent dielectric constants. *Journal of Plasma Physics*, 25(3):175–179, 1987.

[2] F.F. Chen. *Introduction to Plasma Physics and Controlled Fusion*, volume 1. Springer, New York, second edition, 2006.

[3] F. Haas, L. G. Garcia, J. Goedert, and G. Manfredi. Quantum ion-acoustic waves. *Physics of Plasmas*, 10:3858–3867, 2003.

[4] A. K. Sundaram, P. K. Kaw, and S. K. Sharma. Kinetic theory of parametric excitation of acoustic waves in piezoelectric semiconductors. *Pramana*, 2:304–311, 1974.

[5] S. Ghosh and Pragati Khare. Effect of density gradient on the acousto-electric wave instability in ion-implanted semiconductor plasmas. *Journal of Physics*, 109, 2006.

[6] N. Nimje, S. Dubey, and S. Ghosh. Parametric interaction in acousto-optical semiconductor plasma in the presence of hot carriers. *Chinese Journal of Physics*, 49:901–906, 2011.

[7] S. Ghosh and S. Dixit. Modulational instability of a laser beam in a piezoelectric material with strain dependent dielectric constant. *Physics Letters A*, 118(7):354–356, 1986.

[8] Nico F. Declercq, Joris Degrieck, and Oswald Leroy. Sound in biased piezoelectric materials of general anisotropy. *Annalen der Physik*, 14(11–12):705–722, 2005.

[9] M. Raghuwanshi and S. Dixit. To study of quantum effect on modulational instability of laser beam using ferroelectric semiconductors. *International Journal of Scientific & Engineering Research*, 11(4), 2020.

[10] S.S. Sharma and S.P. Singh. Acoustic wave propagation in materials with strain dependent dielectric constant. *Revue de Physique Appliquée*, 12(10):1695–1697, 1977.

[11] K.V. Allamraju and K. Srikanth. Effect of heat on the performance of novel energy harvester by using pzt-5h transducers. *Procedia Engineering*, 144:668–673, 2016.

[12] D.J. Shin, S.-J. Jeong, C.-E. Seo, K.-H. Cho, and J.-H. Koh. Multi-layered piezoelectric energy harvesters based on pzt ceramic actuators. *Ceramics International*, 41:S686–S690, 2015.

[13] N. Yadav, S. Ghosh, and A. Agrawal. Parametric interactions of acoustic waves in semiconductor quantum plasmas with strain dependent dielectric constants. *IOP Conference Series: Journal of Physics: Conference Series*, 1823:012019, 2019.

[14] B. Tiwari, T. Babu, and R.N.P. Choudhary. Effect of manganese doping on dielectric characteristics of lead zirconate titanate of different zirconium/titanium ratios. *IOP Conference Series: Materials Science and Engineering*, 764:012029, 2020.

[15] P. K. Shukla and B. Eliasson. Nonlinear aspects of quantum plasma physics. *Physics-Uspekhi*, 53(1):51–76, 2010.

[16] T.S. Ramadoss and S. Ramakrishna. *Nanotechnology for Energy Sustainability*. Wiley Publishers, 1st edition, 2017.

[17] H. Sahoo et al. Quantum and relativistic effects on the kdv and envelope solitons in ion-plasma waves. *IEEE Transactions on Plasma Science*, 50(6):1469–1475, 2022.

[18] Shilpi. Study of quantum-electron acoustic solitary structures in fermi plasma with two temperature electrons. In *Springer Proceedings in Complexity*, pages 63–83. 2022.

[19] C. Das. Semi-lagrangian method to study nonlinear electrostatic waves in quantum plasma. *IEEE Transactions on Plasma Science*, 50(6):1579–1584, 2022.

[20] D. Mahanta. Impact of ion pressure anisotropy in collisional quantum magneto-plasma with heavy and light ions. *East European Journal of Physics*, (3):143–159, 2024.

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