Ion Acoustic Solitary Wave Formation in a Warm, Unmagnetized Dusty Plasma With Electron Inertia

Rafia Khanam[1], Satyendra Nath Barman[2], and Deepsikha Mahanta[3]*

[1,3] *Department of Mathematics, Gauhati University, Guwahati-781014, Assam, India.*
[2] *B. Borooah College, Guwahati-781007, Assam, India.*
E-mail:mahanta2017@gmail.com
(Received: Nov 25, 2024, Revised: Jan 16, 2025, Accepted: Jan 26, 2025, Published: Mar 30, 2025)

Abstract: The formation of ion-acoustic solitary waves in an unmagnetized plasma with ions, negatively charged warm dust grains, and electrons under the pressure variation has been theoretically and numerically investigated. The governing set of normalized fluid equations are reduced to the Korteweg-de Vries (KdV) equation by the reductive perturbation method to obtain the potential wave amplitude. Ion-acoustic compressive and rarefactive solitons of amplitudes are reported due to the variation of dust to ion density ratio r for fast ion-acoustic modes. The inclusion of electron inertia with pressure variation of the spaces not only significantly modifies the basic features of dust ion-acoustic solitons but also introduces a new parametric regime shown to exist.

Keywords: Solitary waves, KdV equation, Solitons, Negative dust, Electron inertia, Unmagnetized plasma.

I Introduction

A productive area of study for solitary waves in plasma is space, a glamorous and alluring laboratory of nonlinear composition. There are two kinds of dust grains in space. The polarity of plasmas is opposite: (1) large grains are negatively charged, and (2) small grains are positively charged. Due to the fact that heavy dust moves more slowly than ions and electrons, which are accessories in the new time scales, the dust acoustic waves are a very low-frequency, longitudinal compressional wave that involves the moving dust particles. Many researchers have examined the occurrence of ion-acoustic solitary waves in dust charge particle plasma systems in the past few decades, both theoretically [1–8] and experimentally [9,10]. Dusty plasma is found in space, for example, in interstellar clouds, planetary rings, comets, and nebulae. It is possible to create fusion experiments, plasma processing reactions, and other laboratory experiments in a lab setting. Since dusty plasma nonlinear waves are present in many different locations, including rings, the magnetosphere, asteroid zones, thin-film coatings, plasma crystals, etc., they are one of the most intriguing research topics in recent plasma physics. Dusty plasmas have different waves than regular plasmas, and when different kinds of dust-charged grains are present in a plasma, several different wave modes are created. These include the dust ion-acoustic mode [11,12], dust-acoustic mode [13,14], dust-lattice mode [15], dust Berstain-Green-Kruskal mode [16], Shukla-Verma mode [17], and others. These modes are producing new and important results. Furthermore, new wave modes arise when heavy charged particles are present. In lab investigations, the dust ion-acoustic waves have been detected [12,18]. According to theory, Rao et al. [13] initially reported the presence of dust acoustic solitary waves with exceptionally low phase velocity in an unmagnetized dusty plasma. Mendoza-Briceno et al. [19] explore how the temperature of the dust fluid and the nonthermal distribution of ions significantly alter the characteristics of the large amplitude electrostatic solitary structures. Barman and Talukdar [20] investigated electron inertia and nonlinear ion-acoustic waves in a heated, dusty plasma. They discovered both compressive and rarefactive solitons throughout this work. Dust ion acoustic Korteweg-de Vries (KdV) and modified Korteweg-de Vries (mKdV) solitons in dusty plasmas with varying temperatures were compared by Kalita and Das [21]. In the presence of stationary dust, they discovered both compressive and rarefactive Korteweg-de Vries (KdV) solitons of intriguing character that are composed of ions and electrons as well as pressure variations in both components. The weak frequency dust-acoustic solitary waves in electromagnetic and electrostatic waves are theoretically researched by [22,23] and by experts in dust-ion acoustics [24–27].

In this work, we have used the reductive perturbation approach to study ion-acoustic solitary waves in a three-component plasma made up of ions, negatively charged mobile dust, and electrons. Several plasma factors' distinctive effects on the nature and generation of ion-acoustic solitons under various novel circumstances are discussed. The paper is structured as follows: Section 1: 'Introduction'; Section 2: 'Basic Equations governing the dynamics of motion'; Section 3: 'Derivation of KdV Equation and Its Solutions'; and Section 4: 'Results and Discussions'. At the conclusion, "references" are provided.

II Basic Equations Governing the Dynamics of Motion

The three-component collisionless ion-acoustic wave propagation in a heated, unmagnetized plasma including ions, electrons, and negative dust grains has been examined. We also take entire species' pressure variations into account. The fluid normalized equations that follow provide the fundamental equations governing the plasma dynamics of ion-acoustic waves of motion and continuity for such a plasma model.

$$\frac{\partial n_i}{\partial x} + \frac{\partial (n_i u_i)}{\partial x} = 0 \tag{1}$$

$$\frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial x} + \frac{\sigma}{n_i}\frac{\partial p_i}{\partial x} = -z_d \frac{\partial \varphi}{\partial x} \tag{2}$$

$$\frac{\partial u_i}{\partial t} + u_i \frac{\partial p_i}{\partial x} + 3p_i \frac{\partial u_i}{\partial x} = 0 \tag{3}$$

$$\frac{\partial n_d}{\partial t} + \frac{\partial (n_d u_d)}{\partial x} = 0 \tag{4}$$

$$\frac{\partial u_d}{\partial t} + u_d \frac{\partial u_d}{\partial x} + \frac{\sigma}{n_d\, Q}\frac{\partial p_i}{\partial x} = -\frac{z_d}{Q}\ \frac{\partial \varphi}{\partial x} \tag{5}$$

$$\frac{\partial u_d}{\partial t} + u_d \frac{\partial p_d}{\partial x} + 3p_d \frac{\partial u_d}{\partial x} = 0 \tag{6}$$

$$\frac{\partial n_e}{\partial t} + \frac{\partial (n_e u_e)}{\partial x} = 0 \tag{7}$$

$$\frac{\partial u_e}{\partial t} + u_e \frac{\partial u_e}{\partial x} = \frac{z_d}{Q'}\left(\frac{\partial \varphi}{\partial x} - \frac{1}{n_e}\right)\frac{\partial p_e}{\partial x} \tag{8}$$

$$\frac{\partial u_e}{\partial t} + u_e \frac{\partial p_e}{\partial x} + 3p_e \frac{\partial u_e}{\partial x} = 0 \tag{9}$$

$$\frac{\partial^2 \varphi}{\partial x^2} = n_e + z_d n_d - n_i \tag{10}$$

The suffixes i, d, and e in this context stand for positively charged ions, negatively charged dust grains, and electrons, respectively; The ratio of dust grain to positive ion mass is $Q = m_d/m_i \gg 1$, while the ratio of electron to positive ion mass is $Q' = m_e/m_i$. With T_i/T_d and z_d being the amount of elementary charges present on the dust particle, $\sigma = T_i/T_e$ is the ion to electron temperature ratio. The unperturbed electron number density, n_{e0}, normalizes the particle number densities, n_i, n_d, and n_e in the aforementioned equations; The ion-acoustic speed $c_s = \sqrt{k_b T_e/m_i}$ determines the velocities u_i, u_d and u_e; the characteristic ion pressure $k_b n_{e0} T_i$ determines the pressures p_i, p_d and p_e; the time t is determined by the inverse of the characteristic ion plasma frequency $\omega_{p_i}^{-1} = \sqrt{m_i/4\pi n_{e0} e^2}$; the distance x is determined by the Debye length $\lambda_{D_e} = \sqrt{k_b T_e/4\pi n_{e0} e^2}$; the electron pressure p_e is determined by the characteristic electron pressure $p_{e0} = n_{e0} k_b T_e$; and the electric potential φ is determined by $(k_b T_e/e)$, where T_e is the characteristic electron temperature and k_b is the Boltzmann constant.

III Derivation of KdV Equation and Its Solutions

We have examined the propagation of ion-acoustic waves in a warm, unmagnetized plasma including ions, electrons, and negative dust grains. To describe the propagation of the small amplitude ion-acoustic waves, we apply the reductive perturbation approach to the fundamental set of equations (1)–(10). In order to do this, we present a novel stretched variable as

$$\xi = \epsilon^{\frac{1}{2}}(x - Ut), \quad \tau = \epsilon^{\frac{3}{2}}t \tag{11}$$

Where ϵ is a small non-dimensional parameter that measures the strength of nonlinearity and U is the phase velocity of the waves. Now, we expand the flow variables asymptotically about the steady equilibrium state in terms of ϵ as follows:

$$\begin{aligned}
n_i &= n_{i0} + \epsilon^1 n_{i1} + \epsilon^2 n_{i2} + \epsilon^3 n_{i3} + \dots \\
n_d &= n_{d0} + \epsilon^1 n_{d1} + \epsilon^2 n_{d2} + \epsilon^3 n_{d3} + \dots \\
n_e &= 1 + \epsilon^1 n_{e1} + \epsilon^2 n_{e2} + \epsilon^3 n_{e3} + \dots \\
u_i &= \epsilon^1 u_{i1} + \epsilon^2 u_{i2} + \epsilon^3 u_{i3} + \dots \\
u_d &= \epsilon^1 u_{d1} + \epsilon^2 u_{d2} + \epsilon^3 u_{d3} + \dots \\
u_e &= \epsilon^1 u_{e1} + \epsilon^2 u_{e2} + \epsilon^3 u_{e3} + \dots \\
p_i &= p_{i0} + \epsilon^1 p_{i1} + \epsilon^2 p_{i2} + \epsilon^3 p_{i3} + \dots \\
p_d &= p_{d0} + \epsilon^1 p_{d1} + \epsilon^2 p_{d2} + \epsilon^3 p_{d3} + \dots \\
p_e &= 1 + \epsilon^1 p_{e1} + \epsilon^2 p_{e2} + \epsilon^3 p_{e3} + \dots \\
\varphi &= \epsilon^1 \varphi_1 + \epsilon^2 \varphi_2 + \epsilon^3 \varphi_3 + \dots
\end{aligned} \tag{12}$$

Equating the coefficients of the first lowest-order of ϵ using the transformation (11) and expansions (12) in equations (1)–(9) and the boundary constraints $n_{i1} = n_{d1} = 0, u_{i1} = u_{d1} = 0$ and $\varphi_1 = 0$ at $|\xi| \to \infty$, we obtain

$$\begin{aligned}
n_{i1} &= \frac{n_{i0} z_d \varphi_1}{L}, \quad n_{d1} = \frac{-n_{d0} z_d \varphi_1}{M}, \quad n_{e1} = \frac{-z_d \varphi_1}{N} \\
u_{i1} &= \frac{U z_d \varphi_1}{L}, \quad u_{d1} = \frac{-U z_d \varphi_1}{M}, \quad u_{e1} = \frac{-U z_d \varphi_1}{N} \\
p_{i1} &= \frac{3 p_{i0} z_d \varphi_1}{L}, \quad p_{d1} = \frac{-3 p_{d0} z_d \varphi_1}{M}, \quad p_{e1} = \frac{-3\varphi_1}{N}
\end{aligned} \tag{13}$$

where $L = U^2 - 3\sigma, M = U^2 Q - 3\sigma, N = U^2 Q^{'} - 3z_d$.

Again, using (11) and (12) in equation (10), we obtain

$$1 + z_d n_{d0} - n_{i0} = 0 \tag{14}$$

$$\Rightarrow \frac{n_{d0}}{n_{i0}} = r, \quad \frac{1}{n_{i0}} = 1 - z_d r \tag{15}$$

Where $r = \frac{n_{d0}}{n_{i0}}$ is negative to positive ion number density ratio, and $n_{i0} = p_{i0}, n_{d0} = p_{d0}$.

$$n_{e1} + z_d n_{d1} - n_{i1} = 0 \tag{16}$$

Using the expressions of n_{i1}, n_{d1} and n_{e1} from (13) and (14) in (16), the expression for phase speed U is found as

$$\frac{-z_d \varphi_1}{U^2 Q' - 3z_d} - \frac{n_{d0} z_d^2 \varphi_1}{U^2 Q - 3\sigma} - \frac{n_{i0} z_d \varphi_1}{U^2 - 3\sigma} = 0 \tag{17}$$

Since equation (17) is quadratic in U, it depicts two different kinds of ion-acoustic modes that propagate at various phase velocities, namely

$$U^2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{18}$$

where

$$\begin{aligned}
a &= Q(1 - rz_d) + Q^{'} rz_d + QQ^{'} \\
b &= -3\sigma(Q+1)(1 - rz_d) - (3\sigma Q^{'} + 3z_d) rz_d - 3\sigma Q^{'} - 3Qz_d \\
c &= 9\sigma \left\{ \sigma(1 - rz_d) + rz_d^2 + z_d \right\}
\end{aligned}$$

Again, equating the coefficients of second higher-order terms of ϵ from (1)–(7), we get the followings,

$$\frac{\partial n_{i1}}{\partial \tau} - U\frac{\partial n_{i2}}{\partial \xi} + n_{i0}\frac{\partial n_{i2}}{\partial \xi} + \frac{\partial (n_1 u_1)}{\partial \xi} = 0 \tag{19}$$

$$\begin{aligned}
n_{i0}\frac{\partial u_{i1}}{\partial \tau} - Un_{i0}\frac{\partial u_{i2}}{\partial \xi} - Un_{i1}\frac{\partial u_{i1}}{\partial \xi} + n_{i0} u_{i1}\frac{\partial u_{i1}}{\partial \xi} + \sigma\frac{\partial p_{i2}}{\partial \xi} \\
+ n_{i0} z_d \frac{\partial \varphi_2}{\partial \xi} + n_{i1} z_d \frac{\partial \varphi_1}{\partial \xi} = 0
\end{aligned} \tag{20}$$

$$\frac{\partial p_{i1}}{\partial \tau} - U\frac{\partial p_{i2}}{\partial \xi} + u_{i1}\frac{\partial p_{i1}}{\partial \xi} + 3p_{i0}\frac{\partial u_{i2}}{\partial \xi} + 3p_{i1}\frac{\partial u_{i1}}{\partial \xi} = 0 \tag{21}$$

$$\frac{\partial n_{d1}}{\partial \tau} - U\frac{\partial n_{d2}}{\partial \xi} + n_{d0}\frac{\partial n_{d2}}{\partial \xi} + \frac{\partial (n_{d1} u_{d1})}{\partial \xi} = 0 \tag{22}$$

$$Qn_{d0}\frac{\partial u_{d1}}{\partial \tau} - UQn_{d0}\frac{\partial u_{d2}}{\partial \xi} - UQn_{d1}\frac{\partial u_{d1}}{\partial \xi} + Qn_{d0}u_{d1}\frac{\partial u_{d1}}{\partial \xi} + \sigma\frac{\partial p_{d2}}{\partial \xi} - n_{d0} z_d\frac{\partial \varphi_2}{\partial \xi} - n_{d1} z_d\frac{\partial \varphi_1}{\partial \xi} = 0 \tag{23}$$

$$\frac{\partial p_{d1}}{\partial \tau} - U\frac{\partial p_{d2}}{\partial \xi} + u_{d1}\frac{\partial p_{d1}}{\partial \xi} + 3p_{d0}\frac{\partial u_{d2}}{\partial \xi} + 3p_{d1}\frac{\partial u_{d1}}{\partial \xi} = 0 \tag{24}$$

$$\frac{\partial n_{e1}}{\partial \tau} - U\frac{\partial n_{e2}}{\partial \xi} + \frac{\partial n_{e2}}{\partial \xi} + \frac{\partial (n_{e1} u_{e1})}{\partial \xi} = 0 \tag{25}$$

$$Q^{'}\frac{\partial u_{e1}}{\partial \tau} - UQ^{'}\frac{\partial u_{e2}}{\partial \xi} - UQ^{'} n_{d1}\frac{\partial u_{d1}}{\partial \xi} + Q^{'} u_{e1}\frac{\partial u_{e1}}{\partial \xi} + z_d\frac{\partial p_{d2}}{\partial \xi} - z_d\frac{\partial \varphi_2}{\partial \xi} - n_{e1} z_d\frac{\partial \varphi_1}{\partial \xi} = 0 \tag{26}$$

$$\frac{\partial p_{e1}}{\partial \tau} - U\frac{\partial p_{e2}}{\partial \xi} + u_{e1}\frac{\partial p_{e1}}{\partial \xi} + 3\frac{\partial u_{d2}}{\partial \xi} + 3p_{e1}\frac{\partial u_{e1}}{\partial \xi} = 0 \tag{27}$$

Similarly, from equation (10), we obtain

$$\frac{\partial^2 \varphi}{\partial \xi^2} = n_{e2} + z_d n_{d2} - n_{i2} \tag{28}$$

$$\Rightarrow \frac{\partial^3 \varphi}{\partial \xi^3} = \frac{\partial n_{e2}}{\partial \xi} + z_d \frac{\partial n_{d2}}{\partial \xi} - \frac{\partial n_{i2}}{\partial \xi} \tag{29}$$

Eliminating $u_{i2}, u_{d2}, u_{e2}, p_{i2}, p_{d2}$ and p_{e2} from (19)–(27) and then using the first order terms from (13), we get,

$$\frac{\partial n_{i2}}{\partial \xi} = \frac{2Un_{i0}z_d}{(U^2-3\sigma)^2}\frac{\partial \varphi_1}{\partial \tau} + \frac{n_{i0}z_d}{U^2-3\sigma}\frac{\partial \varphi_2}{\partial \xi} + \frac{3n_{i0}z_d^2(U^2-3\sigma)}{(U^2-3\sigma)^3}\varphi_1\frac{\partial \varphi_1}{\partial \xi} \tag{30}$$

$$\frac{\partial n_{d2}}{\partial \xi} = \frac{2QUn_{i0}z_d}{(U^2Q-3\sigma)^2}\frac{\partial \varphi_1}{\partial \tau} + \frac{n_{d0}z_d}{U^2Q-3\sigma}\frac{\partial \varphi_2}{\partial \xi} + \frac{3n_{d0}z_d^2(QU^2+\sigma)}{(QU^2-3\sigma)^3}\varphi_1\frac{\partial \varphi_1}{\partial \xi} \tag{31}$$

$$\frac{\partial n_{e2}}{\partial \xi} = \frac{-2Q^{'}Uz_d}{(U^2Q'-3\sigma)^2}\frac{\partial \varphi_1}{\partial \tau} - \frac{z_d}{U^2Q'-3\sigma}\frac{\partial \varphi_2}{\partial \xi} + \frac{3z_d^2(U^2Q^{'}+z_d)}{(U^2Q'-z_d)^3}\varphi_1\frac{\partial \varphi_1}{\partial \xi} \tag{32}$$

Now, putting the values of $\frac{\partial n_{i2}}{\partial \xi}$, $\frac{\partial n_{d2}}{\partial \xi}$ and $\frac{\partial n_{e2}}{\partial \xi}$ from (30)–(32) into the equation (29) and using the relations (14) and (16), the KdV equation is obtained as

$$\frac{\partial \varphi_1}{\partial \tau} + \alpha\varphi_1\frac{\partial \varphi_1}{\partial \xi} + \beta\frac{\partial^3 \varphi}{\partial \xi^3} = 0 \tag{33}$$

where the nonlinear coefficients α and dispersion coefficient β are given by

$$\alpha = \frac{3z_d(L+4\sigma)M^3N^3 - 3z_d^2 r(M+4\sigma)L^3N^3 - 3(1-rz_d)(N+4z_d)L^3M^3}{2ULMN\{M^2N^2 + Qrz_dL^2N^2 + Q'(1-rz_d)M^2L^2\}} \tag{34}$$

$$\beta = \frac{(1-rz_d)L^2M^2N^2}{2Uz_d\{M^2N^2 + Qrz_dL^2N^2 + Q'(1-rz_d)M^2L^2\}} \tag{35}$$

In this model of plasma, we observed that the nonlinear ion-acoustic soliton exist when $U^2 \neq 3\sigma, 3\sigma/Q, 3z_d/Q^{'}$ subject to the condition $z_d < 1/r$ and $r \leq 0.5$.

The Korteweg-de Vries (KdV) equation (33) can be solved for stationary solitary waves by introducing a new transformation, $\eta = \xi - V\tau$, where V is the soliton speed in the linear η-space. By integrating this transformation into the partial differential equation (33), the solitary wave solution can be obtained as

$$\phi_1 = \varphi_0 \mathrm{sech}^2\left(\frac{\eta}{\Delta}\right) \tag{36}$$

Where $\varphi_0 = 3V/\alpha$ is the amplitude of the ion-acoustic soliton and $\Delta = \sqrt{4\beta/V}$ is the width of the ion-acoustic soliton.

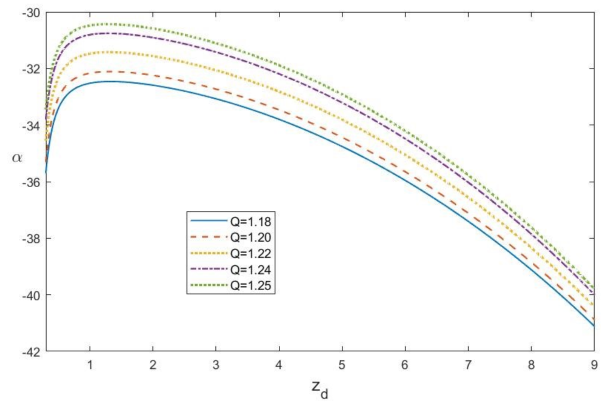

Figure 1: Variation of nonlinear coefficient α versus z_d for different values of Q.

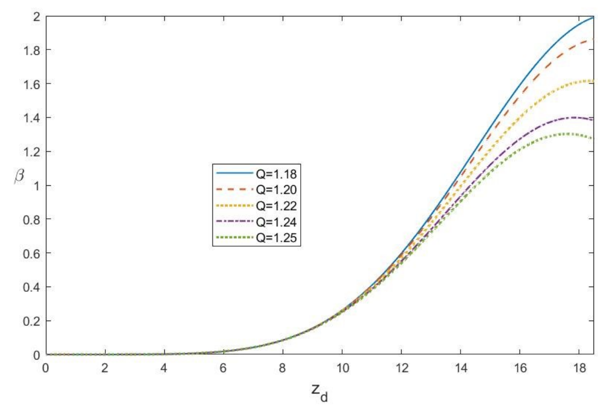

Figure 2: Variation of dispersion coefficient β versus z_d for different values of Q.

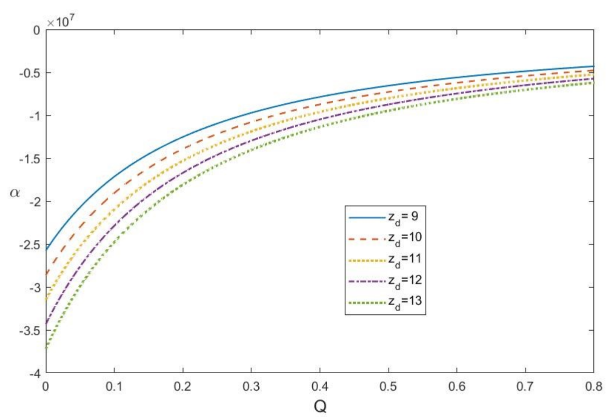

Figure 3: Variation of nonlinear coefficient α versus Q for different values of z_d.

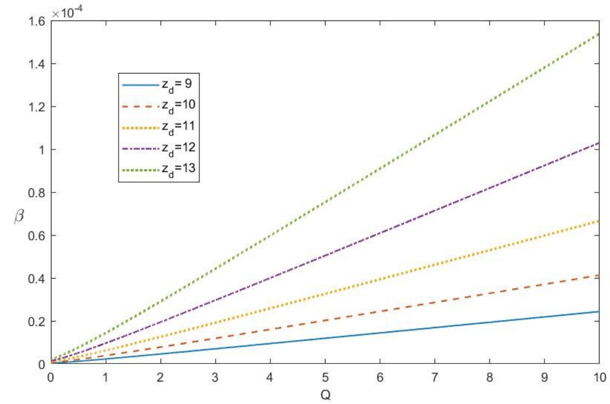

Figure 4: Variation of dispersion coefficient β versus Q for different values of z_d.

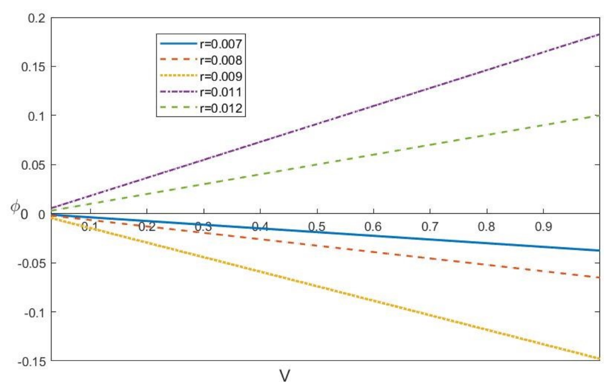

Figure 5: Variation of amplitude (φ_0) versus V for different values of r.

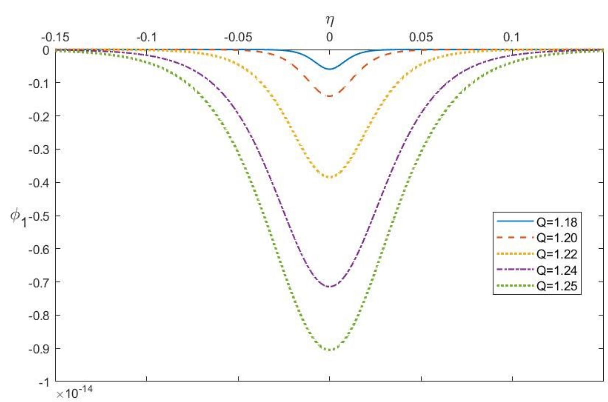

Figure 6: The graph of $\phi_1(\eta)$ versus η for different values of Q.

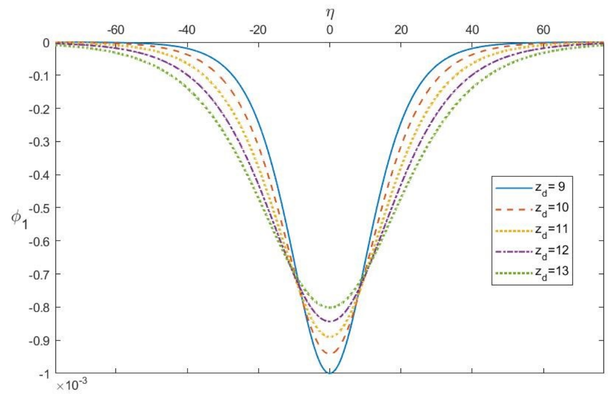

Figure 7: The graph of $\phi_1(\eta)$ versus η for different values of z_d.

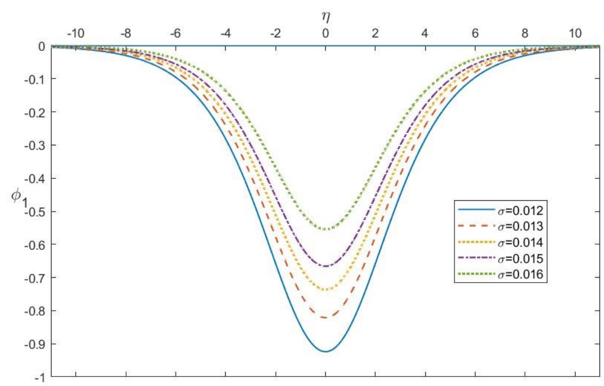

Figure 8: The graph of $\phi_1(\eta)$ versus η for different values of σ.

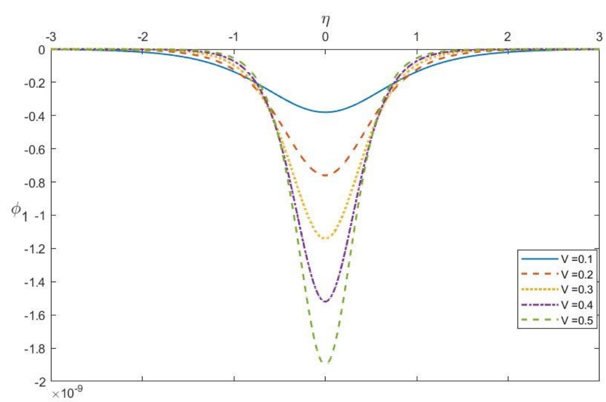

Figure 9: The graph of $\phi_1(\eta)$ versus η for different values of V.

IV Results and Discussion

In this model, we have used the KdV equation to establish the ion acoustic solitons in a dusty plasma made up of positive ions, negative dust grains, and electrons under pressure change. According to our research, only the fast ion-acoustic mode exists because of electron inertia.

In Figure 1, we showed the variation of nonlinear coefficient α and Figure 2, dispersion coefficient β versus z_d for different values of dust grain to positive ion mass ratio $Q = 1.18, 1.20, 1.22, 1.24, 1.25$ and fixed few values of $\sigma = 0.1, Q^{'} = 0.0005446623, r = 0.001, V = 0.1$. Where we found that the growth of rarefactive soliton α (Figure 1) is nonlinearly decreases as the direction of wave propagation approaches the direction of z_d as increasing values of Q. On the other hand, the absolute linear growth of nonlinear coefficient is due to inactive role of z_d. Of course, β (Figure 2) is of the compressive soliton is linearly increases with increasing values of as z_d well as Q.

In Figure 3, we observe that variation of nonlinear coefficient α and Figure 4 dispersion coefficient β versus Q with different values of $z_d = 9, 10, 11, 12, 13$ and fixed few values of $\sigma = 0.01, Q^{'} = 0.0005446623, r = 0.1, V = 0.1$. Where we found that α (Figure 3) is rarefactive raises linearly at the amount of increase of z_d as well as Q and for the growth values of rarefactive soliton sharply increases with dust grain to positive ion mass ratio Q. Again, β (Figure 4) exhibit compressively increase as increasing values of z_d as well as Q. The results obtained in Figure 3 seem to differ with the results obtained in Figure 4 for the number of elementary changes (z_d) cases.

Figure 5 shows that for dust to ion density ratios $r = 0.007, 0.008, 0.009, 0.011$, and 0.012, compressive and rarefactive Korteweg-de Vries (KdV) solitons exist. The consequences of positive and negative possibilities are shown, respectively. With fixed $\sigma = 0.1, Q^{'} = 0.0005446623, Q = 1000$, and $z_d = 100$, compaction occurs when $r > 0.01001$ and rarefaction occurs when $r < 0.01001$. According to this, r is crucial for the presence of rarefactive and compressive soliton feature. After a certain $r^*(\approx 0.01001)$ indicates an unexplained region with the same other characteristics, the rarefactive soliton's character transforms into a compressive soliton.

The solitary wave potential $\phi_1(\eta)$ as a function of η for various values of $Q = 1.18, 1.20, 1.22, 1.24$, and 1.25 when $\sigma = 0.1, r = 0.001, Q^{'} = 0.0005446623, z_d = 100$, and $V = 0.1$ is depicted in Figure 6. We observed that as Q increases, the rarefactive ion-acoustic soliton's amplitude and width increase as well. The solitary wave potential $\phi_1(\eta)$ as a function of η for various values of $z_d = 9, 10, 11, 12$ and 13 when $\sigma = 0.01, r = 0.1, Q = 1000, Q^{'} = 0.0005446623$ and $V = 0.1$ is depicted in Figure 7. As z_d grows, we observe that the rarefactive ion-acoustic soliton's width somewhat increases and its amplitude slightly decreases.

Next, as illustrated in Figure 8–9, we analyze how the solitary wave potential $\phi_1(\eta)$ given in (36) varies with η. The effects of negative possibilities are displayed with lower differences of $\sigma = 0.012, 0.013, 0.014, 0.015, 0.016$ and $V = 0.1, 0.2, 0.3, 0.4$, and 0.5, respectively. Here, we found that the rarefactive ion-acoustic soliton propagates and that, with rising values of σ and fixed $r = 0.001, z_d = 100, Q = 1000, Q^{'} = 0.0005446623$, and $V = 0.1$, the amplitude of the solitary pulse reduces as the increasing values of the pulse gradually expands (Figure 8). Furthermore, with increasing values of σ and fixed $r = 0.001, z_d = 100, Q = 1000, Q^{'} = 0.0005446623$, and $V = 0.1$, we discovered that the rarefactive ion-acoustic soliton's amplitude progressively grows while its width gradually reduces (Figure 9).

V Conclusion

The dynamical characteristics of the propagation of small amplitude DIA solitary waves in an unmagnetized plasma model, which includes electrons, ions, and negatively charged dust grains, have been computationally examined in this work. The reductive perturbation method is used to determine the KdV equation and to acquire their solitary wave solutions. We discuss numerically the effects of physical parameters like dust grain to positive ion mass ratio Q, electron to positive ion mass ratio $Q^{'}$, dust particle z_d and σ is the ion to electron temperature ratio over DIA solitary wave potentials represented by the KdV equations, when the results observed in this study can be contracted as follows:

1. There are two different types of wave modes identified in the current plasma model: slow acoustic modes and fast DIA acoustic modes. However, when extracting KdV equations, only rapid DIA modes are taken into consideration.

2. The dust to ion density ratio r exhibits both rarefactive and compressive Korteweg-de Vries (KdV) solitons.

3. There are only rarefactive Korteweg-de Vries (KdV) solitons that show the σ, V, z_d and Q effects.

References

[1] C. K. Goertz. Dusty plasmas in the solar system. *Reviews of Geophysics*, 27(2):271–292, 1989.

[2] T. G. Northrop. Dusty plasmas. *Physica Scripta*, 45(5):475, 1992.

[3] P. Bliokh, V. Sinitsin, V. Yaroshenko, and D. A. Mendis. Dusty and self-gravitational plasmas in space. *Physics of Plasmas*, 4:68–68, 1997.

[4] E. C. Whipple. Potentials of surfaces in space. *Reports on Progress in Physics*, 44(11):1197, 1981.

[5] M. Horanyi and D. A. Mendis. Trajectories of charged dust grains in the cometary environment. *Astrophysical Journal, Part 1*, 294:357–368, 1985.

[6] F. Verheest. *Waves in Dusty Plasma.* Kluwer Academic, Dordrecht, 2000.

[7] M. Horanyi and D. A. Mendis. The effects of electrostatic charging on the dust distribution at halley's comet. *Astrophysical Journal, Part 1*, 307:800–807, 1986.

[8] P. K. Shukla and A. A. Mamun. *Introduction to Dusty Plasma Physics.* CRC Press, 1st edition, 2001.

[9] D. A. Mendis and M. Rosenberg. Some aspects of dust-plasma interactions in the cosmic environment. *IEEE Transactions on Plasma Science*, 20(6):929–934, 1992.

[10] D. Asoka Mendis and M. Rosenberg. Cosmic dusty plasmas. *Annual Review of Astronomy and Astrophysics*, 32:419–463, 1994.

[11] P. K. Shukla and V. P. Silin. Dust ion-acoustic wave. *Physica Scripta*, 45(5):508, 1992.

[12] A. Barkan, N. D'Angelo, and R. L. Merlino. Experiments on ion-acoustic waves in dusty plasmas. *Planetary and Space Science*, 44(3):239–242, 1996.

[13] N. N. Rao, P. K. Shukla, and M. Y. Yu. Dust-acoustic waves in dusty plasmas. *Planetary and Space Science*, 38(4):543–546, 1990.

[14] A. A. Mamun. Arbitrary amplitude dust-acoustic solitary structures in a three-component dusty plasma. *Astrophysics and Space Science*, 268:443–454, 1999.

[15] A. Homann, A. Melzer, S. Peters, and A. Piel. Determination of the dust screening length by laser-excited lattice waves. *Physical Review E*, 56(6):7138, 1997.

[16] M. Tribeche and T. H. Zerguini. Small amplitude bernstein–greene–kruskal solitary waves in a thermal charge-varying dusty plasma. *Physics of Plasmas*, 11(8):4115–4122, 2004.

[17] P. K. Shukla and R. K. Varma. Convective cells in nonuniform dusty plasmas. *Physics of Fluids B: Plasma Physics*, 5(1):236–237, 1993.

[18] B. C. Kalita and R. Kalita. Implicit role of cairns distributed ions and weak relativistic effects of electrons in the formation of dust acoustic waves in plasma. *Journal of Plasma Physics*, 82(2):905820201, 2016.

[19] Cesar A. Mendoza-Briceno, S. M. Russel, and A. A. Mamun. Large amplitude electrostatic solitary structures in a hot non-thermal dusty plasma. *Planetary and Space Science*, 48(6):599–608, 2000.

[20] S. N. Barman and A. Talukdar. Propagation of ion-acoustic waves in a warm dusty plasma with electron inertia. *Astrophysics and Space Science*, 334(2):345–349, 2011.

[21] B. C. Kalita and S. Das. Comparative study of dust ion acoustic korteweg–de vries and modified korteweg–de vries solitons in dusty plasmas with variable temperatures. *Journal of Plasma Physics*, 83(5):905830502, 2017.

[22] A. A. Mamun, R. A. Cairns, and P. K. Shukla. Solitary potentials in dusty plasmas. *Physics of Plasmas*, 3(2):702–704, 1996.

[23] B. Sahu and R. Roychoudhury. Electron acoustic solitons in a relativistic plasma with nonthermal electrons. *Physics of Plasmas*, 13(7), 2006.

[24] A. A. Mamun and P. K. Shukla. Linear and nonlinear dust-hydromagnetic waves. *Physics of Plasmas*, 10(11):4341–4349, 2003.

[25] C. R. Choi, C.-Mo Ryu, N. C. Lee, and D.-Y. Lee. Ion acoustic solitary waves in a dusty plasma obliquely propagating to an external magnetic field. *Physics of Plasmas*, 12(2), 2005.

[26] T. Saha, P. Chatterjee, and M. R. Amin. Nonlinear ion acoustic waves in a magnetized dusty plasma in the presence of nonthermal electrons. *Zeitschrift für Naturforschung A*, 64(5-6):370–376, 2009.

[27] M. G. Abdelsalam. Quantifying 3d post-accretionary tectonic strain in the arabian–nubian shield: superimposition of the oko shear zone on the nakasib suture, red sea hills, sudan. *Journal of African Earth Sciences*, 56(4-5):167–178, 2010.